PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Effects of Changes of Temperature on the Modulus of Torsional Rigidity of Metal Wires

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Phil. Trans. R. Soc. Lond. A 1905 **204**, 1-55
doi: 10.1098/rsta.1905.0001

PHILOSOPHICAL TRANSACTIONS.

I. *The Effects of Changes of Temperature on the Modulus of Torsional Rigidity of Metal Wires.*

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Communicated by Professor J. J. Thomson, *F.R.S.*

Received March 1,—Read April 28, 1904.

The effect of heat upon the modulus of torsional rigidity was first studied by Kupffer in 1848,* who investigated the changes in rigidity by observations on the torsional vibrations of wires. This method is the one which has since found most favour with experimenters, being easier in manipulation and, perhaps, also capable of greater accuracy than the statical method of experimenting. Kupffer experimented on wires of iron, platinum, silver, copper, and gold, but the range of temperature employed was only that furnished by the varying temperature of the room—never more than 10° C. In a later paper† an account is given of some experiments on wires of copper, steel, and brass at the temperature of the room and at that of boiling water. The coefficient of diminution of the rigidity modulus was assumed to be constant between these temperatures, and was calculated from the periods of the torsional oscillations.

Napiersky‡ repeated Kupffer's experiments with wires of iron, brass, and silver, using a range of temperature of only a few degrees.

Kohlrausch and Loomis§ experimented on wires of iron, copper, and brass at temperatures between 15° C. and 100° C., and came to the conclusion that for these metals the rate of alteration of the rigidity modulus with the temperature was not constant but increased with the temperature. Their results showed that the rigidity modulus at t° C. could be represented fairly accurately in the form

$$n_t = n_0(1 - \alpha t - \beta t^2),$$

n_0 being the value of the modulus at 0° C. obtained by exterpolation.

* 'Mém. de l'Acad. de St. Pétersb.,' VI. ser., t. V., p. 233, 1853.
† 'Mém. de l'Acad. de St. Pétersb.,' VI. ser., t. VI., p. 397, 1857.
‡ 'Pogg. Ann.,' Ergsbd. iii., p. 351, 1853.
§ 'Pogg. Ann.,' 141, p. 481, or 'Amer. Journ. Sci.,' vol. 50, 1870.

PISATI,* in a series of extremely thorough experiments, measured the torsional rigidity of various wires over a range of temperatures from 20° C. to 300° C. He, too, found that the decrease of rigidity per degree rise of temperature increased, at first fairly rapidly, with the temperature, but more slowly afterwards, the rate being practically constant above 190° C. The results of his experiments he expressed in a formula of the type

$$n_t = n_0 - \alpha t + \beta t^2 - \gamma t^3.$$

TOMLINSON† found that constant values of the rigidity modulus could only be obtained after the wire under test had been very carefully annealed. He worked at four temperatures between 0° and 100° C., and his results are given in a formula similar to that adopted by KOHLRAUSCH and LOOMIS.

Experiments have also been made by KATZENELSOHN,‡ by GRAY, BLYTH, and DUNLOP,§ and by SUTHERLAND,|| the latter confining his researches to the softer metals, lead, zinc, tin, and magnesium, which had been neglected by former experimenters.

SCHAEFER¶ worked at the ordinary temperature of the room, and at the temperature of boiling liquid air. He used the statical method of experimenting by observations on the torsional deflections of a wire under a constant couple. The experiments were extended to a large number of elements, but observations were taken at two temperatures only, and the rate of alteration of rigidity with temperature was assumed to be constant between them. The main object of this research was to confirm the law that if the various metals are arranged in order of ascending temperature coefficients of the torsional modulus, they are also in order of increasing coefficients of expansion and of diminishing melting-points.

The rigidity of copper and steel at − 186° C. has also been investigated by BENTON.**

In view of the different results obtained by these observers, it appeared desirable to repeat the determination of the rigidity modulus at different temperatures, taking greater precautions to obtain a constant uniform temperature in the wire under test, timing the torsional oscillations with greater accuracy, and paying especial attention to the purity and physical condition of the wires used. The experiments described in the following paper were performed on wires of various pure metals and some others, all of approximately the same diameter and length. The method employed was a dynamical one, the torsional oscillations of the wire under test being timed by a method of coincidences capable of great exactness. Some of the observations made

* 'Nuovo Cimento,' ser. III., vol. 1, p. 181; vol. 2, p. 137; vol. 4, p. 152; vol. 5, pp. 34, 135, 145; 1877–79.

† 'Roy. Soc. Proc.,' vol. xl., p. 343, 1886.

‡ 'Beibl. zu den Ann.,' XII., p. 307, 1888.

§ 'Roy. Soc. Proc.,' vol. 67, p. 180, 1900.

|| 'Phil. Mag.,' [5], vol. 32, p. 31, 1891.

¶ 'Ann. d. Phys.,' V., p. 220, 1901; IX., pp. 665, 1124, 1902.

** 'Phys. Rev.,' vol. xvi., p. 17, 1903.

in the course of this work yielded what seemed to be interesting information as to the internal viscosity of the wires used. This is recorded and compared with similar observations by other experimenters. The rest of the paper is divided into the following sections:—(I.) Description of apparatus, etc.; (II.) Account of the experiments; (III.) Summary of results and comparison with those of other observers; (IV.) Determination of the coefficients of expansion of the wires.

PART I.

Description of Apparatus, etc.

In these experiments the wire under test and the vibrator were both enclosed in a heating jacket, the temperature of which could be regulated as required. This arrangement was the result of many experiments to find the best method of overcoming the various difficulties connected with obtaining a constant uniform temperature in the wire. Some of these preliminary experiments will be referred to later, and the reasons for rejecting old forms of the apparatus will be given.

The part of the heating jacket surrounding the wire consists of two concentric tubes (H, fig. 1). The outer one is a 3-inch brass tube, and is covered externally with layers of cotton wool to a thickness of an inch all round. The inner tube is of copper $\frac{1}{8}$ inch in thickness and 1 inch in internal diameter. This is joined to two short lengths of wider copper tube at the top and bottom, in order to admit the clamps for the wire. The inner tube of the jacket projects about $1\frac{1}{2}$ inches beyond the outer one at the top, and is there held firmly in a vertical position by a strong wrought-iron clamp, I, fixed to the wall of the laboratory, and projecting about 6 inches from it. The heater is also supported by a larger clamp which grips the outer cylindrical tube just above the box containing the vibrator.

The vibrating plate hangs inside the double-walled copper box F. The floor of this, requiring to be rigid, is made of $\frac{1}{8}$-inch metal, but the other walls are not so thick. The edges were, unfortunately, soldered together instead of being brazed, and this caused many weeks of delay, the apparatus having to be repeatedly resoldered to prevent leaking at the higher temperatures, due to unequal expansions tearing some of the joints asunder. As will be seen from the figure, the double walls of this box are in direct communication with those of the cylindrical part of the heater, so that the steam by which the jacket is heated flows through the two. It is, of course, necessary to have a door to the front of the box. This also has double walls and is fed with steam by means of a side tube from the cylindrical part of the heater. The waste-pipe from the door joins that from the bottom of the box, as is shown in the figure. These pipes are made of $\frac{1}{2}$-inch "composition" tubing and are joined on to the door by means of brass "unions," a thin washer made of sheet asbestos being used to make the joint tight. The door and the front of the box on which it fits have projecting flanges made of thick copper, and the two are screwed up tightly together by 8 screws, a sheet of thin asbestos cardboard being placed between. The internal measurements of the box are 7 inches square by 5 inches deep, and the thickness

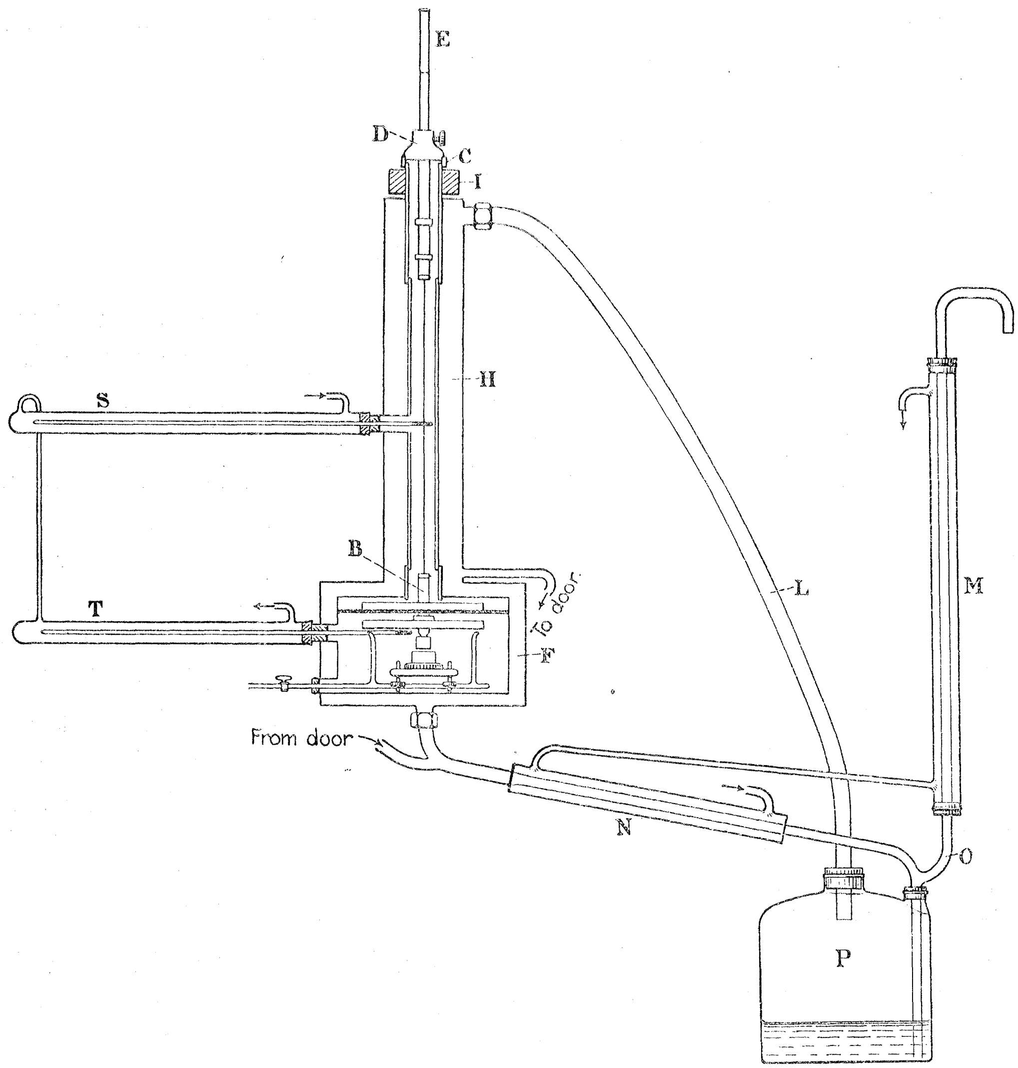


Fig. 1.

between the two walls is about 1 inch. In the centre of the door is a small window of optically worked glass through which the mirrors used in the timing arrangements are viewed.

To the upper end of the inner copper tube of the cylindrical part of the heater the brass collar C is screwed, and in this works the brass head D, through the centre of which passes the $\frac{1}{2}$-inch steel rod E, the lower end of which is made into a clamp to hold the wire. This clamp is shown on a larger scale in fig. 2, and was made in the following manner. The end of the rod was turned down about $\frac{1}{8}$ inch and a steel collar made to fit it. About $\frac{3}{4}$ inch from the end the groove H was made perpendicular to the axis of the rod, which was then slit down to this point with a fine saw. A very thin piece of metal was then inserted in the slit, the collar placed on and screwed up tightly by the steel screw S, and a hole bored down the axis of the rod of the same size as the wire to be used (No. 20, B.W.G.). In this way a neat clamp is formed which grips the wire all around its circumference at a perfectly definite point, viz., the ends of the jaws.

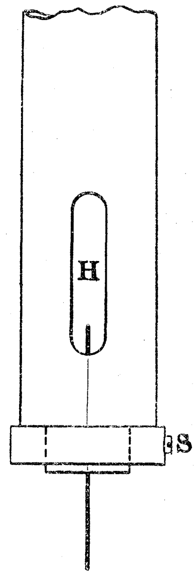


Fig. 2.

The lower end of the wire is held in a clamp exactly similar to that at the top. This clamping rod has a screw thread at its lower end by which the vibrator is fixed to it, being screwed home against a projecting flange. Below the screw thread, the rod was turned down to $\frac{1}{4}$ inch in diameter and one half cut away, so as to leave a plane face for affixing the mirror used in timing the vibrations. This mirror was held in a light brass frame which could be fixed in a perfectly definite position on the clamping rod by means of two small screws.

The vibrating plate hangs about 1 inch below the top of the box, and above it a ring of gunmetal is supported on two horizontal glass rods going from side to side. This ring is for determining the moment of inertia of the vibrator, and can easily be lowered by hand on to the plate. The plate is a circular disc of gunmetal $\frac{3}{8}$ inch thick and 5 inches in diameter. The outer edge of the upper face is turned down about $\frac{1}{16}$ inch. The ring exactly fits on to the groove so formed, and there is thus no difficulty in placing it centrally on the plate.

In order to set the plate in vibration without swinging it like a pendulum bob, the following device was adopted. Two glass jets inclined at 45 degrees to the vertical in opposite directions are fixed underneath the plate, and about $\frac{1}{4}$ inch below it, one at each end of a diameter. These two jets are connected together at the bottom of the box by a glass tube which passes out through the side, and is joined outside to 3 or 4 yards of fine "composition" tubing, which passes along the walls of the laboratory and ends on a small table beside the observing telescope. Here it is joined by means of a piece of indiarubber tubing to a small glass funnel, and by depressing this in a dish of mercury a wind is sent through the jets which puts a couple on the plate and rotates it. By properly timing the instants at which the funnel is lowered into the mercury (by watching through the telescope), the amplitude of vibration of the plate is quickly increased.

The Heating Arrangements.

Observations of the periods of vibration of the wires tested were made in general at five temperatures, viz., at the temperature of the room, about 16° C., at 35° C., 55° C., 75° C., and 100° C., and also in some cases at 126° C. These temperatures were indicated by two delicate mercury thermometers, one of which had its bulb in the cylindrical copper tube close to the wire and at the middle of its length; and the other protruded horizontally into the copper box containing the vibrator, and passed a few millimetres below the plate, the bulb being at the middle of the box (see S and T, fig. 1). These thermometers were graduated to ·1° C., and readings to ·01° C. could readily be made. In order that the readings of the thermometers might not be affected by the varying temperature of the room, the parts of the stems protruding from the jacket were enclosed in glass tubes about an inch in diameter, through which a constant stream of water was kept flowing. The temperature of this stream was ascertained by means of separate thermometers, one in each water-jacket.

In the observations at laboratory temperatures the heating jacket was connected to a water tap, and a rapid stream of water allowed to flow through it. It would then, in general, take three or four hours for the temperatures, as indicated on the thermometers, to become constant. The upper one, as would be expected, arrived at a steady state long before the lower, but in no case were observations taken until both had attained a constant temperature. The necessity for this precaution is at once seen, for if the temperature of the thermometer is varying, then it is most probably different from that of the plate or wire, as the case may be. Also, if the plate is at a different temperature from the wire, there must be a flow of heat taking place along the wire, which consequently cannot be at a uniform temperature.

The other temperatures were all obtained by using the vapours of various liquids boiling under atmospheric pressure. The liquids used were ether (35°), acetone (55°), methylated spirit (75°), water (100°), and amyl alcohol (126°). In order that the composition of the liquid, and therefore its boiling-point, might not vary, a reflux arrangement was used. This is shown in fig. 1. The vapour is driven from the large copper boiler P, through the tube L, into the top of the heating jacket. It passes out at the bottom, and is condensed in the water condenser N, the condensed liquid running back into the boiler. The boiler is open to the atmosphere through the tube O, which is also surrounded by a water-jacket, and serves as a safety tube. The tube L is a 1-inch "composition" tube, and is fitted to the heating jacket by means of a brass union joint, a washer made of thin sheet asbestos being used to make the joint tight. The tube was well wrapped up in cotton wool, this being covered externally with a cylinder of asbestos cardboard—as was also the whole of the cotton wool round the heating jacket—in order to prevent it from taking fire in case of an accident.

In the first form of the apparatus, the wire alone was surrounded by the heating jacket, and the vibrator hung inside a wooden box, the temperature of which was indicated by a thermometer passing close to the plate. Happening one day to put the thermometer at a further distance from the plate than usual, I was struck by the difference in the temperature indicated. This led me to experiment with several thermometers at different distances from the plate, and the results showed that the plate was much hotter than the rest of the enclosure. The seriousness of this is at once evident, for not only were the temperatures which had been taken to be those of the vibrator too low, but heat must have been continually flowing from the wire, which, therefore, could not have been at the temperature of the jacket enclosing it.

It was first attempted to stop this leakage of heat from the wire by interposing a rod of some non-conducting material between the vibrator and the wire. Some experiments were made on the conductivities of various materials, and finally slate was selected as being the best from a workable and non-conducting point of view. A rod of slate, 4 inches long and $\frac{1}{2}$ inch diameter, was fitted between the vibrator plate and the clamp which held the wire. Even with this arrangement the plate warmed up a little, about 2° C., when the heating jacket was at 100° C. In order to see how this affected the distribution of temperature along the wire, a copper wire was suspended in the heater, and to this were soldered three nickel wires at different points along its length, thus giving three pairs of thermo-electric junctions, by means of which any temperature differences could be detected. The results were surprising. A point, $\frac{1}{2}$ inch from the lower clamp and 4 inches inside the heating jacket, was found to be at a temperature $3\frac{1}{2}$° C. below that of the centre of the wire, while at a point on the wire, 6 inches inside the heater, this difference was 2·8° C. These experiments indicate that the only way to obtain a uniform temperature in the wire is by enclosing the whole vibrating system in the heating jacket. The interposition of non-conducting material reduces the temperature differences, but does not entirely do away with them.

As the heat had been leaking out of the wire through the bottom clamp, it must also have been doing the same through the top one. The top clamp was therefore severed from the bar E (fig. 1), and a piece of slate, 3 inches in length, interposed between the two. The rod was then lowered further inside the heater, and the difference of temperature between the upper end of the wire and the centre was determined. No deflection could be obtained on the galvanometer used, showing that the temperature difference was less than ·01° C. After the new heater described in this paper had been made, the distribution of temperature along the wire was again tested, but no difference could be found between that at the centre and that at either end. The piece of slate interposed in the upper clamping rod unfortunately cracked with the heat after being used for a few weeks, and a similar one of marble was therefore substituted. This was exceedingly difficult to turn owing to its brittleness, but it has successfully withstood the effects of temperature for over a year, while further experiments with the thermal junctions proved it to be as effective as slate in preventing a leakage of heat.

It might here be mentioned, that this research was commenced at the Physical Laboratory of the University of Birmingham. In the first form of apparatus used, the wire experimented on was 110 centims. long. On starting afresh at the Cavendish Laboratory—where most of the work described in this paper was carried out—this size was reduced to 58 centims., the smaller heating jacket thereby required being more convenient, in that it comes to a steady state more quickly and is more easily maintained at a constant temperature.

Corrections to the Thermometer Readings.

The thermometers used were four in number, two ranging from 0° to 100° C., and two from 100° to 200° C., all graduated to ·1°. These were standardised at the National Physical Laboratory, the tests being carried out with the thermometers under exactly the same conditions as obtained in these experiments, viz., in a horizontal position, and with nine divisions in the hot bath, and the rest of the stem surrounded by a water jacket at a known temperature. From the figures supplied by the National Physical Laboratory, curves of corrections were drawn, from which the corrections to the thermometer readings at any temperature could be read off with an accuracy of ·01° C. A small correction had to be applied on account of the difference in the temperature of the water-jacket in these experiments and when the thermometer was standardised. This was found by calculation, taking the coefficient of apparent expansion of mercury in glass as ·00016. The correction on this account was never greater than ·04° C.

The method of heating with the vapours of boiling liquids worked extremely well. The jacket would take from 3 to 4 hours to arrive at a constant temperature, and then neither thermometer would vary by ·05° C. even in the worst cases, and it often happened that both thermometers remained absolutely steady for hours. It is a curious fact that the corrected readings of the two thermometers were rarely the same. The difference was generally less than ·1° C., but sometimes the upper and sometimes the lower one would indicate the higher value. In calculating the results, the temperature of the wire was always assumed to be that of the upper thermometer, while the lower thermometer was taken as indicating the temperature of the vibrator.

Timing the Vibrations by a Method of Coincidences.

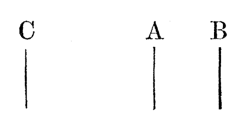
The method used consists in observing the reflections of a vertical flash occurring once a second, in two mirrors, one of which is fixed in position, and the other is attached to the vibrating plate and vibrates with it, swinging just above the fixed mirror and being parallel to it when at rest. The reflections are observed by a telescope, in the field of view of which, in general, two flashes are seen, one always occurring in the same position, and the other appearing in different parts of the field, according to the position of the moving mirror at the instant the flash occurs. If a second signal happens exactly when the two mirrors are parallel, the two flashes coincide, and it is from these "coincidences" that the time of vibration is obtained.

The method of coincidences is usually only applied to the comparison of two nearly equal times, but Professor POYNTING suggested to the author a modification of the method which can be applied to any two periods, even if they are quite different. In order to illustrate this, let us assume that the time of a complete vibration is 4·116 . . . seconds. This has to be compared with the 1 second period of the flashes. Now suppose that the two images of the flash as seen in the telescope have just coincided, and let us call the second at which this coincidence occurred 0. Then after 4 seconds the two mirrors will not be exactly parallel again, and the flashes observed in the telescope will consequently be some distance apart. Since the period is greater than 4 seconds, the moving mirror will not yet have become parallel to the fixed one, and the moving flash will appear to have fallen short of its zero position. If, however, we wait for such a number of seconds, n, as is very nearly an exact multiple of the time of vibration, then at the nth flash the two mirrors will be very nearly parallel, and the flashes will very nearly coincide. Now $9 \times 4{\cdot}116 = 37{\cdot}044$; therefore after 37 seconds, the moving flash will appear very near indeed to the fixed one, for it would take the swinging mirror only ·044 second to become parallel to the fixed mirror. As before, since the multiple of the time of swing is greater than 37 seconds, the flash will appear to have fallen short of its position of rest. Now suppose we go on counting the seconds, calling the flashes next after the 37th, one, two, three, &c., up to 37 and then starting at one again, and so on. Every 37th flash will appear to have lost on the position of the preceding 37th, *i.e.*, to have moved further away from the central fixed flash in the opposite direction to that in which it would appear to be moving if the light were continuous. As this goes on flash number 1 (*i.e.*, the one next after the 37th) will be getting nearer and nearer to the central fixed flash, until after a time it coincides with it. Suppose that when this happens we have counted N sets of 37 seconds. Then it is evident that the mirror must have lagged behind one complete second, for it takes one second longer for it to become parallel to the fixed mirror.

The vibrator therefore makes 9N vibrations in 37N + 1 seconds, and if T is the period of vibration we have

$$9NT = 37N + 1, \quad \text{or} \quad T = \tfrac{37}{9} + \frac{1}{9N}.$$

In general an exact coincidence is of very rare occurrence, and the nearest coincidences are taken. Thus suppose A is the position of the fixed flash as seen in the telescope and that a 37th flash appears at B, and the next 37th at C. Then the mirrors must have been parallel at the fraction AB/BC ($= x$ say) of a period after B, so that the coincidence period is $N + x$ instead



of N, and we have

$$T = \frac{37}{9} + \frac{1}{9(N + x)}.$$

In order to measure AB and BC the telescope used had a fine horizontal scale in the eyepiece, and, as a general illumination of the field of view of the telescope made it difficult to see the flashes, the scale was illuminated by the light from a small rectangular slit in a dark screen placed in front of a lamp. The light was reflected into the telescope by means of a small right-angle prism, a piece of ground glass was placed in front of the slit to make the illumination more uniform, and also a piece of green glass, for it was found that the flashes could be best seen when the scale was illuminated with this colour.

In order to see what degree of accuracy this method is capable of, it must be noticed that an error can only arise in the fraction x, and only an inexcusably careless observer could be ·1 out in the value found If we take N as 20 (a mean value) we see that

$$\frac{1}{9(N + x)} = \frac{1 - x/N}{9N} \text{ approximately};$$

the error occurs in $x/9N^2$ and is therefore not greater than

$$1/(90 \times 20^2) = \cdot000027 \text{ second}.$$

If N is less than 20 it follows that the flashes are "losing" more rapidly, and hence the distances to be measured in the telescope are larger, and the value of x can be more accurately found. If on the other hand N is larger than 20, the distances between two consecutive 37th flashes is smaller, and consequently x is more difficult to measure; but in none of the cases dealt with could the error have been as large as ·1, and it must be observed that the larger N becomes of less importance is the fraction x.

In what has been said above it is assumed that the vibrator is gradually losing on the 37 second period. This is of course not always the case. It may be gaining and making 9 vibrations in a little under 37 secconds and $9(N + x)$ vibrations in $37(N + x) - 1$ seconds; whence

$$T = \frac{37}{9} - \frac{1}{9(N + x)}.$$

In this case each 37th flash appears to gain on the previous 37th, and so to move out from the centre of the field in the opposite direction to that already considered. After N periods it is now the 36th flash which nearly coincides with the fixed one. The coincidence period is of course not always 37. To find it the time of vibration must first be approximately known. This is found by GAUSS's method, using a chronometer. Suppose it to be 4·1154 seconds. Such a number of seconds is required as is nearly an exact multiple of this time. Considering the fractional part only, put $\cdot 1154 \times P = Q$, whence $Q/P = \frac{1154}{10000}$ successive convergents to which are $\frac{1}{8}, \frac{1}{9}, \frac{2}{17}, \frac{3}{26}$. Hence the numbers which multiplied by 4·1154 give products nearest whole numbers are 8, 9, 17, 26. . . The larger the factor the nearer is the product to a whole number, and at first sight it might seem to be of advantage to use the largest to obtain the coincidence period, but taking 26 we have $26 \times 4\cdot 1154 = 107\cdot 0004$, so that in a period of 107 seconds the vibrator will have "lost" ·0004 second, and therefore to lose 1 second it will take 2500 periods of 107 seconds, or over 70 hours! In the present instance 9 is the most suitable factor and gives $9 \times 4\cdot 1154 = 37\cdot 0386$, so that in a period of 37 seconds the vibrator lags behind by ·0386 second, and therefore "loses" 1 second in 16 minutes.

The Flashing Apparatus.

The second signals are given by a clock pendulum, carrying at its lower end a platinum wire, which cuts through a fine stream of mercury each time the pendulum passes its position of rest. This "makes" a circuit, which by means of a relay "breaks" another circuit through the primary of an induction coil, thus giving a flash in a vacuum tube connected to the secondary circuit. The platinum wire is flattened out perpendicularly to the plane of swing of the pendulum. It is 6 millims. long and is screwed into a small brass pin, which fits into a hole in the lower end of the pendulum, being secured in position by a screw. It can easily be removed for cleaning. This brass pin is joined to a fine copper wire running up the back of the wooden pendulum rod and connected at the top to the lower end of the steel strip by which the pendulum is suspended. The pendulum suspension is connected by a wire to a terminal screw outside the clock case.

The stream of mercury issues from a very fine glass jet fixed to the back of the clock case. An adjustment in the position of this jet can be made by means of two large milled-headed screws, so as to direct the stream exactly through the position of the platinum wire when the pendulum is at rest. The jet is at the end of a glass tube which enters the case through a hole in the side, and is bent so as to direct the stream of mercury perpendicularly to the plane of swing of the pendulum. This tube is connected by indiarubber to a reservoir containing some 500 cub. centims. of mercury, at a level of about 30 centims. above the jet. The rate of flow of the mercury can be regulated by a stop-cock. In the glass tube, just before the jet, are two air traps, in order to stop any bubbles of air which might interrupt the mercury stream. A platinum wire, connected outside to a terminal, is also sealed into the tube to make contact with the mercury. The mercury running from the jet is collected in a wooden box from which it can be drawn off and returned to the reservoir.

The clock break is connected in series with two accumulator cells and a contact breaker, shown in fig. 3. The wires from the clock are connected to the terminals of the small electromagnet A. Every second this becomes excited and attracts a soft iron armature, B, which is fixed to the shorter arm of the light brass lever C. This lever is 18 centims. long and the short arm is 2·5 centims. To the longer arm two

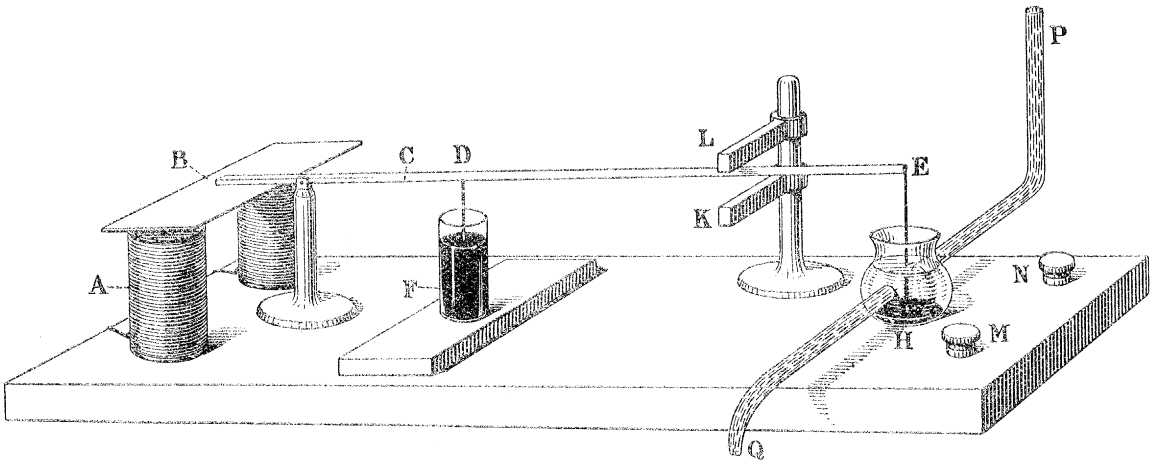


Fig. 3.

platinum wires are brazed, one at D and the other at E. The wire at D dips into a mercury cup, F, and is not drawn out when the magnet is excited. The wire at E dips into another cup of mercury, H, and the position of the lever is so adjusted, by the two stops K and L, that E is drawn well out of the mercury in H when the clock circuit is made. The terminals M and N are connected to the mercury in the cups F and H respectively. Wires from them go to the primary of the induction coil, two accumulator cells being included in this circuit. The contact breaker on the induction coil is tied back so that it cannot work, and hence once a second the primary circuit is broken and a flash occurs in the vacuum tube connected to the secondary.

At first a layer of water was kept over the surface of the mercury in H, but after a time this became very dirty (the "dirt" being probably a platinum amalgam in a very fine state of division) and a skin formed on the surface of the mercury. This led to the circuit sparking two or three times at each break, and caused much trouble, for the result is two or more flashes in the vacuum tube (not separately visible), and therefore two or more in different parts of the field of view of the telescope. In order to overcome this difficulty, the arrangement shown in the figure was used. The cup H was made by blowing a hole in the side of a bulb blown in a glass tube, one end of which was then bent up (P) and the other slightly down (Q). The mercury in H does not come up to the level of these side tubes. P is connected to the stop-cock of a vessel containing water and Q hangs over a bucket. The water flows slowly into H and its level gradually rises until the extra pressure is sufficient to overcome the surface tension of the drop which always hangs from Q. It then runs out with a rush, leaving only a thin layer of water above the surface of the mercury. This is repeated, and the intermittent rush of water is quite effectual in keeping the surface of the mercury clean. The bouncing of the lever on hitting against the stops was another difficulty which caused double sparking. This was overcome by covering the stops with cork.

In order that the platinum wire at the end of the pendulum should make good contact with the mercury, it was occasionally removed, scraped, heated, and amalgamated with sodium amalgam. It was then gently heated again to unamalgamate it. This method—recommended by C. V. BOYS*—was found to give the best results. The wire E on the contact breaker had to be periodically taken out and ground smooth on an oil stone, for the sparking made the end ragged. It was then heated and treated in the manner described above.

In order to prevent sparking at the clock break, a battery of 10 small cells (connected up in sets of two in series and the sets joined up in parallel), consisting of strips of platinum foil in accumulator acid, were placed in parallel across it. These soon become polarised, and their back E.M.F. prevents the current from the accumulators from going through them, but the extra E.M.F. when the clock circuit is broken sends a momentary current through these cells instead of sparking across the air gap.

Several other methods of obtaining second signals were tried with different kinds of clock contact and various forms of relay, but after giving much trouble they were discarded for the present arrangement, which has proved very satisfactory.

I had expected that the stream of mercury would affect the rate of the clock, but on rating it for several days with the mercury running continuously from 10 A.M. till 10 P.M., no difference in the rate could be detected. The clock rate was determined daily by comparing the time with that indicated by the clock in the Cambridge Philosophical Society's Library. This latter clock keeps very good time, and is checked once a week by a man from the observatory. Its weekly rate is about 2 seconds. Twice a day—at 10 A.M. and 6 P.M.—a chronometer was carried across to the library and its time compared with that of the Philosophical Society's clock, and then immediately afterwards with the experimental clock in the Cavendish Laboratory. The rates thus obtained were afterwards corrected on account of the rate of the library clock. The clock used in these experiments was found to keep a constant rate for a week, altering slightly each time it was wound up.

The vacuum tube giving the second signals was fixed vertically at a distance of about 4 metres from the mirrors. The light from it was focussed by means of a glass cylinder full of water on to a slit made by cutting a fine line on the back of a silvered mirror. The images of the slit in the mirrors were viewed by a telescope placed 80 centims. from the mirrors, the slit and telescope being so arranged that the incident and reflected light were nearly normal to the mirrors.

* 'Phil. Trans.,' A, vol. 186, p. 25.

The fixed mirror was supported on a small brass stand resting by three levelling screws on the floor of the heating jacket (see fig. 1). It could be rotated about a vertical axis in order to set the mirror parallel to the one on the vibrator.

The Method of Taking an Observation.

In general, on one day, two sets of observations were taken, the first at the temperature of the tap water, and the second at some higher temperature. In order to get the two done in one day it was necessary to leave the water running through the jacket over night, so that in the morning no time should be wasted in waiting for the temperature to become steady.

A set of observations at the temperature of the tap water having been taken, the heating jacket was emptied and dried by sucking warm dry air through it. It was then connected up to the boiler in the manner already described. As a rule, it would take from 3 to 4 hours for the temperature to become constant, and in the interval the rest of the apparatus was tested and set in order if necessary. The fixed mirror had to be re-set parallel to the moving one at each temperature, for the wire altered its zero position slightly. When the temperature had become constant the plate was set vibrating, the angle through which it moved being observed by placing a candle behind the slit so as to illuminate it continuously. The amplitude of oscillation of the plate was increased until the image of the slit moved over slightly more than 22 divisions of the telescope scale. The plate was then left vibrating for a short time in order to get over all the disturbing effects of air currents. The amplitude of swing having damped down to 22 divisions on the eye-piece scale, the time of a chronometer was noticed, and the temperatures of the thermometers read. This was for a determination of the logarithmic decrement of the oscillations which was made with each observation of the period. When the amplitude had further damped down to 20 divisions (= 15 minutes of arc) the candle was removed from behind the slit and the observations of the period begun. Since it was times of complete vibrations that were required, a coincidence being taken when the moving mirror was parallel to the fixed one, and moving always in the same direction, an error might have arisen through variations in the zero position of the wire during an experiment. In order to eliminate this, two sets of coincidence periods were taken in which the flashes either "lost" or "gained" in opposite directions.

Everything being ready, and the temperature of the thermometers noted, the observer sits down to the telescope and watches until he sees a coincidence is about to happen. He notices one flash is "early" by a certain amount while the 37th (if 37 is the coincidence period) is "late" by another amount. These are noted down He must then go on counting from 1 up to 37 continuously, making a note of each 37th. After a time he will observe that a coincidence in the opposite direction is about to happen, *i.e.*, that some flash—say the 23rd—is moving across the field in the opposite direction to the 37th and will soon be coinciding with the fixed flash. This coincidence is noted in the same way as the former one. Then, if the flashes are "losing," after a time the 37th flash will have lost nearly a second and flash number 1 will be about to coincide. This coincidence is waited for and noted. In the same way later on the 23rd gets a second behind and the 24th coincides. It is only necessary to keep watching the flashes just when a coincidence is expected, and in the intervals between the coincidences the thermometers are read every few minutes. In order to make sure that it has not been omitted to make a note of any 37th flash, the time of the chronometer at which one of these flashes happens near the beginning of the observation is noticed, and also again at the end. From the total time the number can be checked.

After the observation of the period was finished, the candle was again placed behind the slit and the amplitude of vibration again observed. When this became an exact number of divisions of the eye-piece scale the time of the chronometer was noticed. From the times at the beginning and the end, and from the calculated period of vibration, the number of vibrations could be obtained, and hence the logarithmic decrement of the swings calculated.

The Effect of Damping on the Period of Vibration.

I had anticipated having to correct the times of vibration on account of the damping of the amplitude, but in nearly all cases the correction was too small to take into account. If the equation of motion of the disc is

$$\ddot{\theta} + \kappa\dot{\theta} + u^2\theta = 0,$$

the period of damped vibration is given by

$$\mathrm{T} = \frac{2\pi}{\sqrt{u^2 - \frac{1}{4}\kappa^2}} = \frac{2\pi}{\sqrt{u^2 - 4\lambda^2/\mathrm{T}^2}},$$

where λ is the logarithmic decrement of the amplitude of vibration.

Now if I is the moment of inertia of the vibrating system, and if n is the rigidity modulus, l the length, and a the radius of the wire, we have $u^2 = n\pi a^4/2l\mathrm{I}$.

Hence $$\mathrm{T} = \frac{2\pi}{\sqrt{n\pi a^4/2l\mathrm{I} - 4\lambda^2/\mathrm{T}^2}}; \quad \text{so that } n = \frac{8(\pi^2 + \lambda^2)\,\mathrm{I}l}{\mathrm{T}^2\pi a^4}.$$

If now we correct T, the observed period, to T_0, the time of vibration the system would have if there were no damping,

$$n = 8\pi^2\mathrm{I}l/\mathrm{T}_0^{\,2}\pi a^4.$$

Equating these two values of n,

$$\frac{\pi^2 + \lambda^2}{\mathrm{T}^2} = \frac{\pi^2}{\mathrm{T}_0^{\,2}} \quad \text{or} \quad \mathrm{T}_0 = \mathrm{T}\left\{1 - \tfrac{1}{2}\left(\frac{\lambda}{\pi}\right)^2\right\}.$$

Except in the case of aluminium and of the soft metals tin, lead, and cadmium, the value of λ was never greater than ·00115, and it was generally much less than this. Taking $\lambda = \cdot 00115$—the value for a gold wire at 100° C.—the period was 5·4 . . . seconds so that

$$T_0 = 5 \cdot 4 \ldots (1 - \cdot 000{,}000{,}08)$$

or the correction in this case is negligible.

In the case of the soft metals it was found to be impossible to time the vibrations to such a degree of accuracy as was obtained with the other wires, and only in a few instances was it necessary to apply the correction for damping.

The Moments of Inertia of the Vibrators.

The same vibrator was used for observations on wires of copper, iron, steel, silver, platinum, gold, and aluminium. It consisted, as already described, of a circular plate of gunmetal screwed through its centre on to the lower end of a steel rod, the upper end of which was formed into a clamp to hold the wire. The total mass of this vibrator was about 1 kilogramme. Lighter vibrators had to be used in the case of the softer metals. With cadmium a rectangular sheet of brass weighing 100 grammes was substituted for the circular gunmetal disc. This was replaced by a similar sheet of aluminium in the case of tin and of lead. The moments of inertia of these two latter vibrators were found with sufficient accuracy by calculation from their masses and dimensions; but more exact knowledge was required in the case of the circular vibrator on account of the greater precision with which the torsional periods of the harder wires could be determined. The moment of inertia of the gunmetal vibrator was therefore determined experimentally, the wire on which it was suspended being a steel one. Attempts were also made to find the moment of inertia of the vibrating system in the case of other wires, but it was found to be impossible to do so, for in nearly all the wires investigated the rigidity was not a constant quantity, but varied very much even at one temperature, so that determinations made at different times could not be compared. Also in the case of the metals copper, gold, and platinum the extra weight of the ring used in determining the moment of inertia altered the rigidities of the wires very considerably, so that the periods with the ring on could not be compared with those of the vibrator alone on this account. The experiment was tried with each wire, and it was found that the time of vibration of the plate alone, before the ring was put on, was largely different from that after it had been taken off. The figures obtained in the determination of the moment of inertia of this vibrator, and its variation with the temperature, will be given later.

The Lengths and Radii of the Wires.

The wires experimented on were all about 58 centims. long. For the purpose of measuring their lengths, a fine mark had been scribed on the upper clamping rod E (fig. 1), and also another on the lower rod B, beneath the gunmetal plate and just above the mirror. This latter mark could be seen through the glass window in the door of the heating jacket. The distance between these two marks was read every few days by means of a cathetometer. The lengths from the marks to the jaws on each of the rods E and B had previously been measured, and by subtracting these distances, corrected for any difference of temperature, from the total length, the length of the wire at the temperature of the observation was obtained.

The radii of the wires were found by weighing a known length of the wire first in air and then in water. The ends of the wire were filed square with the length, which was then accurately measured. The wire was then wound up in the form of a spiral and its volume found by weighing in air and afterwards in distilled water at a known temperature.

The lengths of the clamping rods, of the wires in the determination of their radii, and the dimensions of the ring used in finding the moment of inertia of the vibrator, were all measured on Professor POYNTING's measuring bench in the Physical Laboratory of the Birmingham University. This consists of a trolley running on a railway underneath two vertical microscopes, which can be placed at any desired distance apart. The microscopes have each a rocking-plate micrometer,* by means of which a length of ·001 millim. can be measured. The article whose length is required is placed under one microscope and a standard metre under the other. The trolley is moved until the desired point on the article is under the first microscope, and the reading of the standard metre under the second is noted. The trolley is then pushed until the other point comes under the first microscope, and the scale is again read through the second. In this way the distance between the two points is accurately determined. The length of the standard metre was corrected to 0°, at which temperature it is certified correct.

In obtaining the dimensions of the moment of inertia ring, two diameters at right angles were measured, and then the ring was turned over and the same two diameters measured from the other side.

The weight of the ring was 293·3900 grammes.

The mean radii were 6·35078 centims. and 5·38859 centims. at 13·90° C. Hence the moment of inertia at 13·90° C. = 10176·13 grm. cm.^{2}

Corrections to the Observed Periods of Vibration.

In order that the observed periods at different temperatures may be comparable, it is necessary that they should be corrected for the increased length and radius of the wire, and also for the expansion of the vibrator, at the higher temperatures. Two other corrections might have been necessary, one for the logarithmic decrement of the

* For description, see POYNTING, 'Phil. Trans.,' A, vol. 182, p. 589, 1891, or "The Mean Density of the Earth," p. 95.

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oscillations, and the other for their amplitude. The former was generally too small to take into account, and the latter was avoided by always dealing with the same amplitude.

The formula connecting the period of vibration with the rigidity and dimensions of the wire, and the moment of inertia of the vibrator is $T = 2\sqrt{2lM\pi/na^4}$, where M is the moment of inertia of the vibrator and n the modulus of rigidity, a being the radius and l the length of the wire. If now α is the coefficient of expansion of the material of the wire, and β that of the material of the vibrator, and if the wire is at $\theta°$, and the vibrator at $\phi°$, we have

$$T_\theta = 2\sqrt{2\pi M_0 (1 + 2\beta\phi)\, l_0 (1 + \alpha\theta) / n_\theta a_0^4 (1 + 4\alpha\theta)},$$

where n_θ is the rigidity modulus at $\theta°$. Hence

$$T_\theta = 2\sqrt{\frac{2\pi M_0 l_0}{n_\theta a_0^4}}(1 + \beta\phi - \tfrac{3}{2}\alpha\theta).$$

In order, therefore, to correct the observed periods to the values they would have if the wire and vibrator retained the dimensions they had at 0°, they must be multiplied by $(1 - \beta\phi + \frac{3}{2}\alpha\theta)$. In the experiments 15° C. was taken as the temperature of comparison, and the observed times were corrected by multiplying by

$$\{1 - \beta(\phi - 15) + \tfrac{3}{2}\alpha(\theta - 15)\}.$$

The value of β and the various values of α for the different wires were determined experimentally. The vibrator, as already stated, consisted of a gun-metal plate with a steel rod through its centre, and the temperature coefficient of the moment of inertia of this system was required. This was found in the following manner, a steel wire, the temperature coefficient of the rigidity modulus of which had already been found, being used to suspend the vibrator :—The vibrations were timed at the ordinary temperature and then the ring was lowered on to the vibrator and the period again determined. The ring was next turned round about a vertical axis through an angle of 60° and the period found again, then turned through a further 60°, and so on, taking six observations in all, the mean of which was taken as the period of vibration of the vibrator + ring. The ring was lastly taken off the plate and the period of the vibrator again found, the mean of this and the first determination being taken as the period of the vibrator alone. Each time the heating jacket was opened to move the ring it had to be left for from one to two hours for the temperature to become constant again. On the following day the temperature of the jacket was raised to 100°, and two sets of observations with the ring on and off were taken. It had been intended to take more, but the personal discomfort involved in removing the ring from the plate whilst hot persuaded me to be content with two. The coefficient of linear expansion of the material of the ring was assumed to be the same as that of a long gun-metal bar, which had been cast at the same time as the ring. The coefficient of expansion of this bar, as well as the coefficients of expansion of the various wires, was determined in the manner described in Part IV. of this paper.

The results of the above observations of periods are given below :—

Period in seconds.	Temperature of wire.	Temperature of vibrator.	Ring.
2·915408	17·16	16·59	off
3·581610	17·53	17·19	on
3·581496	17·27	17·13	turned through 60°
3·581613	17·43	17·42	through 120°
3·581660	17·54	17·54	through 180°
3·581089	16·68	16·30	through 240°
3·581268	16·86	16·42	through 300°
2·915498	17·34	16·95	off

Mean of the periods with the ring on = 3·581456 seconds at 17·22° C. ; 17·00° C.

Mean of the periods with the ring off = 2·915453 seconds at 17·25° C. ; 16·77° C.

The mean period with the ring on must now be corrected to the value it would have if the temperatures had been the same as when the ring was off. Correcting therefore on account of the alteration of rigidity of the wire, and of the dimensions of the wire and plate, we get the corrected time Q = 3·581454 seconds.

Calling the period of the vibrator alone T, and I the moment of inertia of the ring, we have

$$\frac{M}{T^2} = \frac{M + I}{Q^2}, \quad \text{or} \quad M = \frac{IT^2}{Q^2 - T^2}.$$

The value of I at 16·77° C. was calculated from the known value at 13·90° C. and the experimentally determined value of the coefficient of expansion.

Hence M at 16·77° C. = 19992·018 grm. cm.^{2}

The values of M at the higher temperature were calculated in an exactly similar manner, and the results obtained were

From the first experiment, M = 20049·400 grm. cm.^{2} at 99·06° C.

From the second experiment, M = 20049·709 grm. cm.^{2} at 98·78° C.

From each of these values, and that given above for 16·77° C., the coefficient of change of moment of inertia of the vibrator with temperature was found. The values were ·00003540 and ·00003501 ; mean = ·00003520 ; from which β required in the formula for correcting the periods is ·00001760.

PART II.

Account of the Experiments.

The wires experimented on were, except in a few cases, chemically pure, and were supplied by Messrs. Johnson, Matthey & Co., of Hatton Garden, London. They were all of approximately the same length and diameter. In order to get them into comparable conditions, they were carefully annealed before the rigidity determinations were begun. This was done by repeatedly heating with an electric current, the oxidisable wires, iron and steel, being enclosed in an atmosphere of pure dry hydrogen, copper in carbonic acid gas, and the others in air. In each case about a metre of the wire was suspended vertically in a glass tube, a light weight being attached to its lower end. This closed an electric circuit by dipping into some mercury at the bottom of the tube. In the cases of steel and of iron the tube was then repeatedly exhausted and filled with pure dry hydrogen until all the air had been removed. Carbonic acid gas was used instead of hydrogen in the case of copper; gold, silver, platinum and aluminium were heated in air. The wire was then repeatedly heated to a red heat by means of an electric current, each heating lasting for several minutes. The temperature attained was of course highest in the case of those metals which are furthest from their melting points at ordinary temperatures. Platinum was taken almost to white heat, and iron and steel well above the temperature of recalescence. After this treatment, a length of about 60 centims. was cut from the middle of the annealed wire and suspended in the heating jacket. All the specimens used were quite straight and free from kinks, and special care was taken to see that the wire had not been heated to too high a temperature, or subjected to any permanent elongation by the tension to which it was subjected during the annealing. The wires were all bright and free from oxide.

The softer metals, lead, tin, and cadmium, were annealed by suspending them (without any weight attached to their lower ends) in a vertical combustion tube, heated by means of vertical rows of small Bunsen burners. A thermometer was suspended beside the wire, and the temperature was not allowed to exceed two-thirds of the melting point of the metal in question. The temperature was kept up for about an hour, and then the tube was allowed to cool down very gradually. The heating and cooling were repeated many times. Some unannealed wires were also experimented on.

Before giving the results in detail for the different wires, it would be well to mention that in every case examined, with the exceptions of pure copper and steel, it was found that the modulus of rigidity at a constant temperature was not constant, but increased as time went on. This increase was generally very small, and was greater at the higher temperatures than at the ordinary temperature of the laboratory. After repeated heatings for long periods to one temperature, the increase with time at that temperature gradually became smaller and smaller, and in some cases became zero; but if then the wire was heated to a higher temperature and subsequently brought back to the lower one again, the increase with time at that temperature still went on. This effect could not be due to any disturbing cause, such as the oxidation of the surface of the wire or a gradual lengthening under the tension, for these would have the opposite effect, and cause an apparent diminution in the rigidity. The wires were quite bright at the end of the experiments, and the length was measured at intervals and in many cases was quite constant. In the case of gold and the softer metals, however, it increased slightly during the experiments, and the periods obtained were corrected on this account.

Under these circumstances it is evidently impossible to determine accurately the manner in which the rigidity is affected by the change of temperature, for such change is complicated by the varying time effect. It was thought that the most useful information would be obtained by plotting the values of the rigidity modulus against the corresponding temperatures, and drawing a curve through the first points obtained at each temperature. The alteration of rigidity per 1° C. rise of temperature at the ordinary laboratory temperature was then found by drawing the tangent to this curve at 15° C. In this manner a coefficient β was obtained for each wire such that for temperatures in the neighbourhood of 15° C. $n_t = n_{15}(1 - \beta t)$, n_{15} being the value of the modulus at 15° C.

In addition to the determinations of the periods of vibration, observations of the logarithmic decrement of the amplitudes of the oscillations were taken at each temperature, as already described. These will be found among the results which follow. At the end of the rigidity determinations for each wire, a series of observations was usually taken to ascertain the manner in which the logarithmic decrement and the torsional period varied with the amplitude of vibration, amplitudes up to about 10° being used. The main observations for the rigidity determinations were all taken at the constant average amplitude of 14 minutes.

Observations at 126° C. were only taken with a few of the wires, for great trouble was experienced through the heating jacket leaking when heated to this temperature.

In the account which follows, the complete set of curves and tables of results are given for a few only of the metals experimented on. These serve as examples of the method of experimenting and as models of the results obtained.

It would be well to remark here that only in a few instances was more than one specimen of the metal in question experimented on. The values given for the rigidity modulus and its temperature coefficient would be only approximately true for any other specimen, for it is a well-established fact that the values of the rigidity modulus of any substance given by different specimens of the same material are not the same. The cases in which more than one sample was experimented on were those in which the "time effect" was least, as it seemed possible that in these cases the temperature coefficient of the rigidity modulus of different specimens might be more constant than the absolute value. This, however, was not found to be the case.

Copper.

The wire used was made from electrolytically purified copper. The observations, the results of which are given below, were all taken with the apparatus in its final form. I had previously experimented on the same wire in the first form of apparatus, and had ascertained that at a constant temperature the period of torsional vibration, and therefore the rigidity, was constant. The periods given below are each the mean of many observations, they have been corrected for clock rate and for the expansion of the wire and vibrator.

TABLE I.—Copper Wire.

Temperature.	Corrected period in seconds.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Date of observation.
14·47	3·941906	4·37053	July 19
14·52	3·941867	4·37062	,, 21
32·67	3·956394	4·33860	,, 21
33·19	3·956752	4·33780	,, 22
18·41	3·944639	4·36450	,, 23
56·68	3·975418	4·29715	,, 24
18·66	3·944603	4·36455	,, 25
75·53	3·989608	4·26664	,, 26
19·16	3·944711	4·36431	,, 28
95·52	4·005477	4·23290	,, 29
19·68	3·944498	4·36480	,, 30

The numbers in the preceding table show that after the wire has been heated the rigidity at the lower temperature does not come back to its original value, but is slightly greater than it was before the heating. The mean of the first two observations is plotted together with the observations at the higher temperatures in Diagram I.

It will be seen that the points fall on a straight line, showing that the rigidity modulus is a linear function of the temperature. From the diagram the value of n at 15° C. was found to be $4{\cdot}3693 \times 10^{11}$ dynes per sq. centim., and the coefficient β to be ·0003877. The modulus at $t°$ is therefore given by the equation

$$n_t = 4{\cdot}3693 \times 10^{11} \{1 - {\cdot}0003877 (t - 15)\}.$$

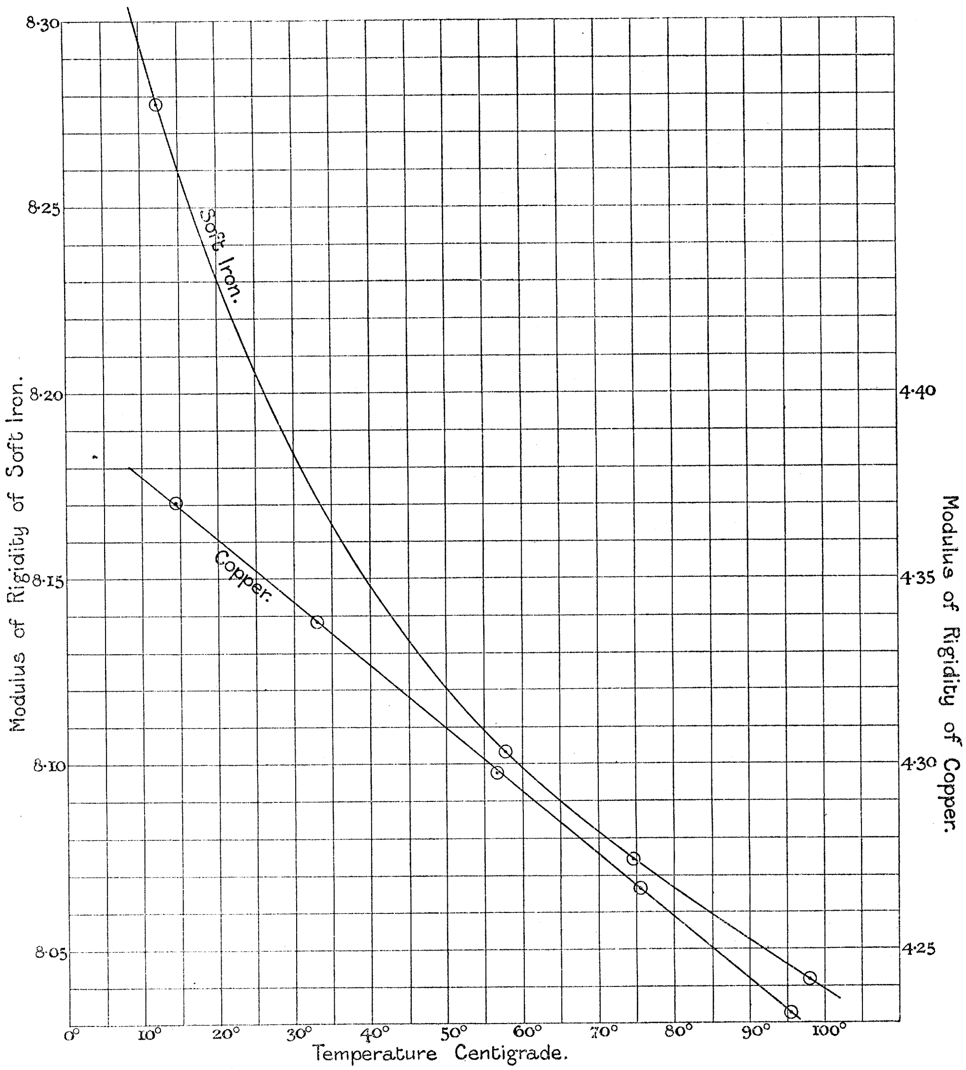

Diagram I. Showing the effect of change of temperature on the modulus of rigidity of soft iron and of copper wire.

The logarithmic decrements of the oscillations at the various temperatures are given in Table II.

TABLE II.—Copper Wire.

Temperature of wire.	Logarithmic decrement of oscillations.	Temperature of wire.	Logarithmic decrement of oscillations.
14·51	·000542	95·54	·000906
33·29	·000636	19·78	·000524
18·56	·000587	124·55	·001620
56·68	·000697	20·00	·000478
18·80	·000564	124·04	·001502
75·53	·000736	18·40	·000414
19·30	·000543		

The values of the logarithmic decrement are plotted against the corresponding temperatures in Diagram II., the points marked being the first observation at the ordinary temperature and the values at the higher temperatures.

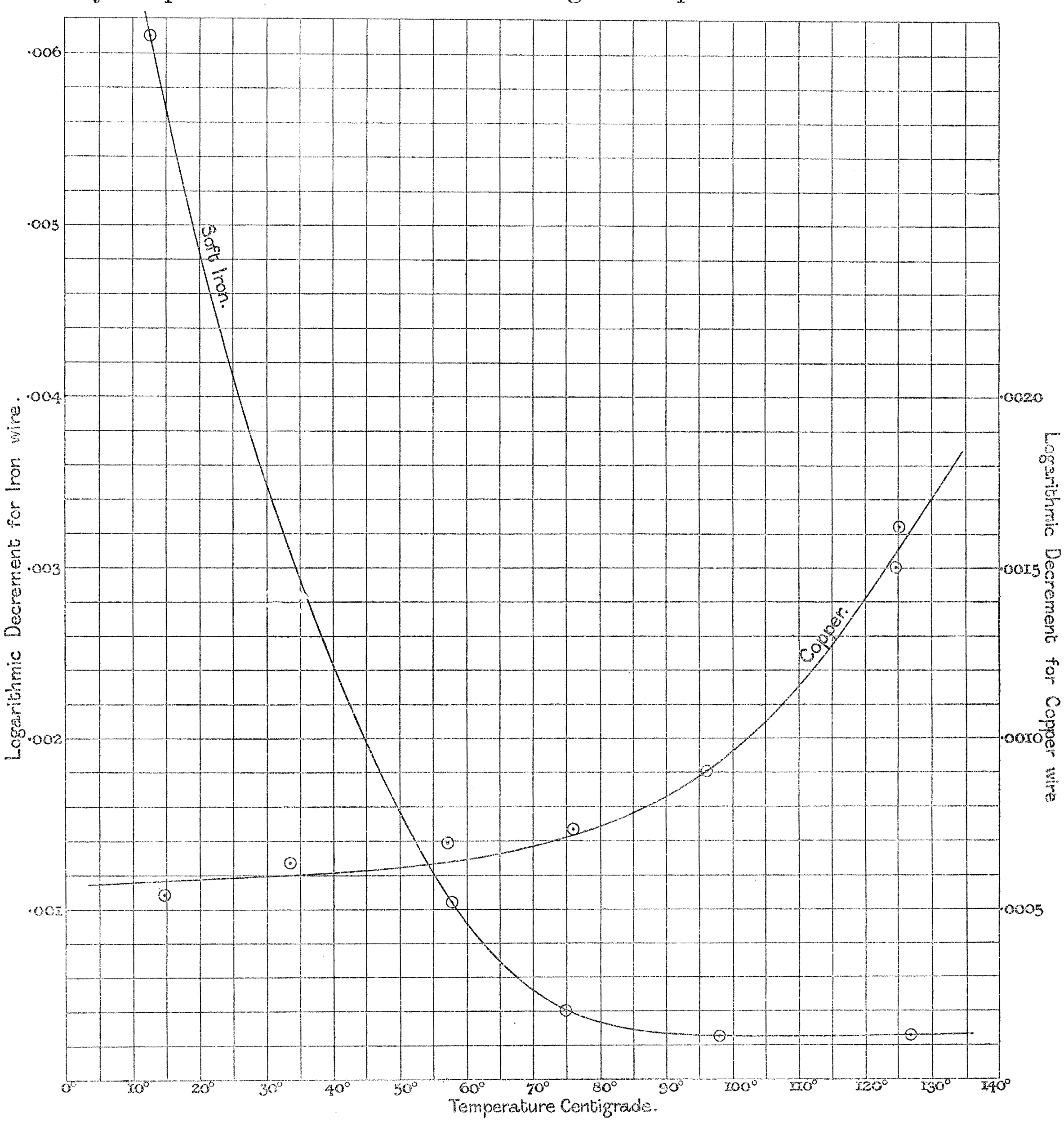

Diagram II. Showing the effect of change of temperature on the logarithmic decrement of the torsional oscillations of iron and of copper wire.

The curve shows that the logarithmic decrement, and therefore the internal viscosity of the wire, increases with the temperature, slowly at first but more rapidly at higher temperatures.

Soft Iron.

This was the first wire in experimenting on which it was noticed that the period of vibration at one temperature was not constant, but gradually diminished as time

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went on. On allowing the wire to cool down and then re-heating to the same temperature on the following day, the period was always very considerably less, the difference being greater the higher the temperature at which the observations were taken. In the table given below, the last column contains the number of hours during which the wire had been maintained at approximately the temperature given. The coincidence period at 33° C. was extremely awkward to work with, and the observations taken at that temperature could not be relied on. They are therefore omitted from the table.

TABLE III.—Soft Iron Wire.

Date.	Temperature.	Corrected Period, in seconds.	Modulus of Rigidity, in 10^{11} dynes per sq. centim.	Time during which temperature had been maintained, in hours.
October 21 . . .	12·43	2·858813	8·27755	—
,, 21 . . .	57·84	2·889403	8·10321	—
,, 22 . . .	17·39	2·863052	8·25306	—
,, 23 . . .	58·94	2·889434	8·10303	—
,, 24 . . .	17·48	2·862964	8·25356	—
,, 24 . . .	74·87	2·894604	8·07411	5·17
,, 24 . . .	74·87	2·894523	8·07457	5·42
,, 24 . . .	74·90	2·894383	8·07535	6·53
,, 25 . . .	18·92	2·863438	8·25083	—
,, 27 . . .	74·62	2·892934	8·08344	6·27
,, 27 . . .	74·65	2·892838	8·08398	6·63
,, 28 . . .	13·40	2·858543	8·27911	—
,, 28 . . .	97·97	2·900418	8·04178	5·17
,, 28 . . .	97·96	2·899936	8·04445	6·12
,, 28 . . .	97·98	2·899769	8·04538	6·30
,, 28 . . .	98·00	2·899695	8·04579	6·47
,, 29 . . .	18·54	2·861014	8·26482	—
,, 29 . . .	98·03	2·897383	8·05863	5·50
,, 29 . . .	98·05	2·897255	8·05934	6·18
,, 29 . . .	98·05	2·897203	8·05964	6·33
,, 29 . . .	98·05	2·897013	8·06069	6·47
,, 29 . . .	98·05	2·896990	8·06082	6·62
,, 30 . . .	18·80	2·858881	8·27716	—
,, 30 . . .	98·05	2·895771	8·06761	5·45
,, 30 . . .	98·09	2·895571	8·06872	6·35
,, 31 . . .	18·26	2·858711	8·27814	—
November 10 . .	12·45	2·854534	8·30239	—
,, 11 . .	12·73	2·854708	8·30137	—
,, 11 . .	98·41	2·890421	8·09751	3·50
,, 11 . .	98·42	2·890309	8·09813	4·50
,, 11 . .	98·42	2·890238	8·09853	6·50
,, 13 . .	98·45	2·890097	8·09932	3·00
,, 13 . .	98·44	2·890081	8 09941	4·33
,, 13 . .	98·64	2·890119	8·09919	6·17
,, 13 . .	98·65	2·890077	8·09943	6·67
,, 14 . .	13·71	2·854634	8·30180	—

In Diagram I. the value of the rigidity modulus from the first determination at each temperature is plotted against the corresponding temperature. It seems probable that the reason this curve is not a straight line is because the rigidity of the wire is continually increasing. If it were possible to obtain the rigidities at the different temperatures all within a short space of time, it might be expected that a straight line would result.

In order to give a clearer idea of the rate at which the rigidity is increasing at the various temperatures, a set of curves has been drawn in Diagram III., in which the values of the rigidity modulus are plotted against times at the different temperatures. From these curves it is seen that the rate of increase of rigidity at

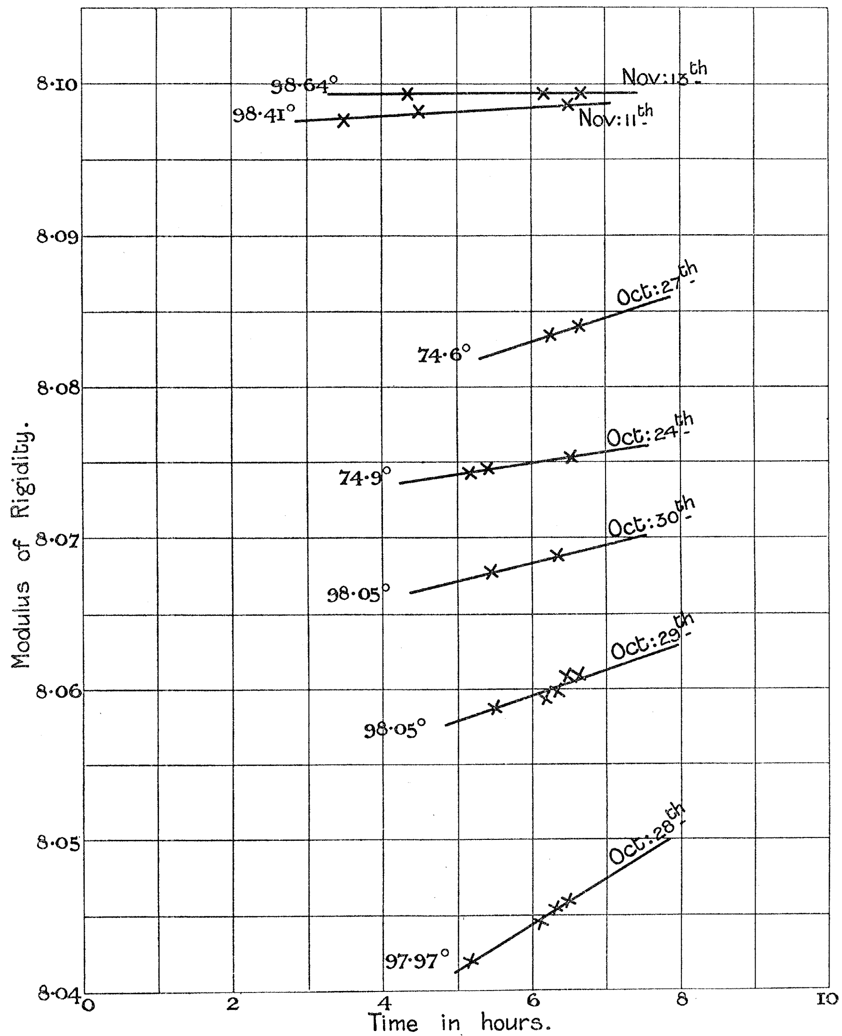

Diagram III. Showing the alteration of the modulus of rigidity of soft iron with time.

one temperature is constant, that it is greater the higher the temperature, but that on repeated heatings to the same temperature the rate of increase at that temperature gets less and less, and finally becomes zero.

From the curve for iron in Diagram I., the value of n at 15° C. was found to be $n_{15} = 8{\cdot}2620 \times 10^{11}$ dynes per sq. centim. and the coefficient $\beta = {\cdot}0007339$.

The values of the logarithmic decrements obtained at the various temperatures are given in Table IV. Each is, in general, the mean of several observations.

TABLE IV.—Soft Iron Wire.

Date.	Temperature of the wire.	Logarithmic decrements of the oscillations.	Date.	Temperature of the wire.	Logarithmic decrements of the oscillations.
October 21 . .	12·49	·006100	October 30 . .	98·07	·000226
" 21 . .	57·77	·001043	" 31 . .	18·02	·002613
" 21 . .	58·06	·001000	November 3 .	126·85	·000259
" 22 . .	17·66	·00678	" 3 .	126·92	·000264
" 23 . .	58·70	·00095	" 5 .	126·84	·000259
" 24 . .	74·84	·000405	" 6 .	127·47	·000253
" 25 . .	19·02	·006236	" 8 .	127·36	·000247
" 27 . .	74·56	·000373	" 10 .	12·50	·001303
" 28 . .	13·48	·005047	" 11 .	12·75	·001270
" 28 . .	97·97	·000257	" 11 .	98·40	·000209
" 29 . .	18·64	·004465	" 13 .	98·56	·000212
" 29 . .	98·02	·000248	" 14 .	13·89	·000884
" 30 . .	18·89	·003380			

From the numbers in the above table it will be seen that the logarithmic decrement decreases with increasing temperature. This is just the opposite to what happened in the case of copper. As a rule repeated heatings diminished the value of the logarithmic decrement. This shows most clearly at the ordinary temperature, where the value of the logarithmic decrement is largest. It is peculiar that after passing 100° C. the value of the logarithmic decrement has attained a minimum and begins to increase with increasing temperature. There is no doubt about this, for a large number of observations were taken at 100° C. and at 126° C., some of those at 100° C. being taken before and some after those at 126° C. This result had also been obtained with another iron wire on which I had previously experimented, using the old form of heating jacket. Another curious point is that the first heating increased the value of the logarithmic decrement at ordinary temperatures, while all subsequent heatings diminished it. This result was also obtained with the first iron wire, and with the copper wire as can be seen in Table II.

In Diagram II. a curve is drawn through the value of the logarithmic decrement from the first observation at each temperature.

When all the observations for finding the rigidity were finished, I investigated whether the time of vibration and the logarithmic decrement varied with the amplitude of swing so long as this was small. The following two results were the means of several determinations :—

Amplitude.	Temperature.	Period.	Logarithmic decrement of oscillations.
12 minutes	11°·48	2·853680	·000788
5 degrees	11·32	2·853865	·000915

The increase in period at the larger amplitude corresponds to a decrease in the rigidity modulus of ·001 × 10^{11} dynes per sq. centim. The value of the logarithmic decrement is seen to increase a little with the amplitude.

The results of experiments on another iron wire, carried out in the old form of apparatus, the wire alone being surrounded by the heating jacket, gave as the modulus of rigidity at 15° C. n_{15} = 7·9656 × 10^{11} dynes per sq. centim., and for the "temperature coefficient" at 15°, β = ·0006047.

The wire used was exactly similar to the one the experiments on which have been described above, being a pure soft iron wire annealed in the same manner. The large difference between the values of β found for the two wires is most probably due to the imperfect method of experimenting used in this case.

Platinum.

A pure platinum wire was used. The periods given in the following table are in all cases the means of several observations.

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Table V.—Platinum Wire.

Date.	Temperature.	Corrected period in seconds.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Time, in hours, during which temperature had been maintained.
November 25 . .	11·90	3·300313	6·46212	—
,, 25 . .	11·85	3·300258	6·46233	—
,, 26 . .	33·47	3·304187	6·44698	4·25
,, 26 . .	33·61	3·304162	6·44707	5·50
,, 27 . .	33·68	3·303872	6·44820	4·00
,, 27 . .	33·92	3·303903	6·44808	5·50
,, 28 . .	13·83	3·299799	6·46413	—
,, 28 . .	58·11	3·308305	6·43093	4·33
,, 28 . .	58·16	3·308246	6·43116	5·80
,, 29 . .	58·49	3·307947	6·43233	—
December 1 . .	16·25	3·299643	6·46474	—
,, 1 . .	73·64	3·310771	6·42136	5·50
,, 1 . .	73·65	3·310567	6·42215	8·83
,, 2 . .	73·85	3·310337	6·42304	6·00
,, 2 . .	73·93	3·310379	6·42288	7·50
,, 2 . .	74·04	3·310315	6·42312	10·67
,, 3 . .	12·31	3·298383	6·46968	—
,, 3 . .	98·92	3·315746	6·40210	—
,, 4 . .	99·13	3·316219	6·40028	5·33
,, 4 . .	99·16	3·316068	6·40086	8·85
,, 5 . .	17·55	3·299458	6·46547	—
,, 9 . .	15·94	3·298789	6·46809	—
,, 10 . .	98·28	3·314998	6·40499	5·57
,, 10 . .	98·31	3·314957	6·40515	6·33
,, 11 . .	14·71	3·298202	6·47039	—
,, 11 . .	125·97	3·322880	6·37464	—
,, 12 . .	126·02	3·322783	6·37501	6·00
,, 12 . .	126·02	3·322639	6·37557	7·65
,, 12 . .	125·92	3·322488	6·37615	10·60
,, 13 . .	16·97	3·298175	6·47050	—
,, 13 . .	98·63	3·314203	6·40807	6·30
,, 13 . .	98·64	3·314299	6·40769	8·30
,, 14 . .	98·63	3·314242	6·40792	6·33
,, 14 . .	98·60	3·314259	6·40785	8·50
,, 14 . .	98·53	3·314217	6·40801	11·67
,, 15 . .	17·03	3·298057	6·47096	—

Between the observations on December 5 and those on December 9 the wire was heated several times to above 100° in an endeavour to obtain the temperature 126°, but owing to the amyl alcohol in the boiler having absorbed some water, the mixture boiled considerably below that temperature. I had therefore to suspend the observations for a few days while I purified the amyl alcohol. During this interval some more observations at 100° were taken, as will be seen from the table. The first observation at each temperature is plotted in Diagram IV.

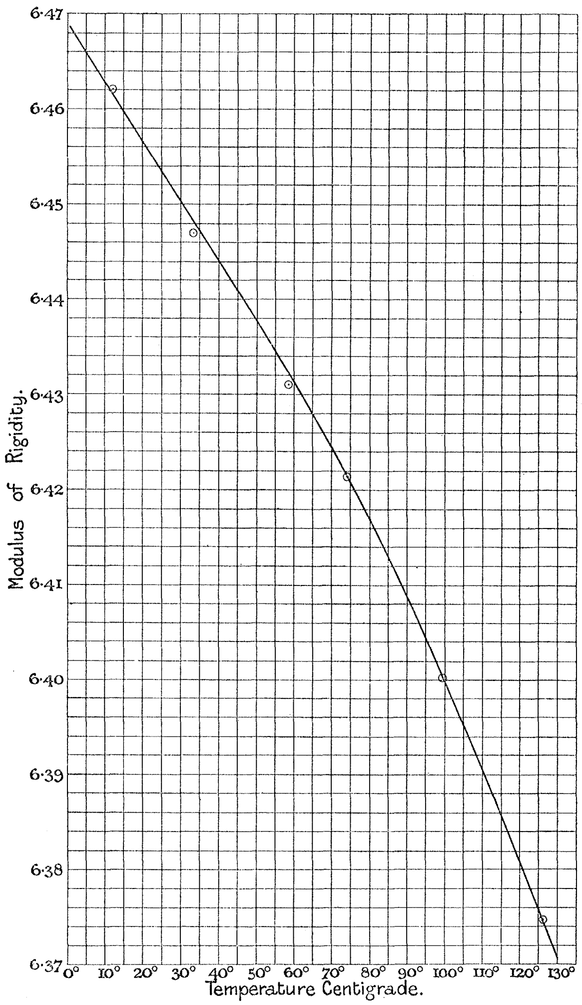

Diagram IV. Showing the effect of temperature on the modulus of rigidity of platinum wire.

The curve in this diagram bends in the opposite manner to that which was obtained for soft iron. It is obvious that the rigidity of the wire falls off with increasing temperature more rapidly than would be required by a linear law. The gradual increase of rigidity with time—which is greater the higher the temperature—is, however, continually acting in opposition to this, so that if it were possible to obtain the rigidity of the wire at different temperatures, taking all the observations within a very short space of time, the deviation from the linear law would be greater than it is in Diagram IV. The shape of this curve for platinum is what one would expect to obtain for a solid near its melting-point; for as the melting-point is passed the rigidity must become zero. (In the case of any solid with a latent heat it is not zero *at* the melting-point.) It is difficult to understand why such a curve is given by platinum—one of the metals which, at ordinary temperatures, is furthest away from its melting-point.

The modulus of rigidity of platinum at 15° C. was found to be $n_{15} = 6{\cdot}4600 \times 10^{11}$

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dynes per sq. centim., and the "temperature coefficient" $\beta = \cdot 0001018$. The alteration of rigidity with time can be easily seen from Diagram V.

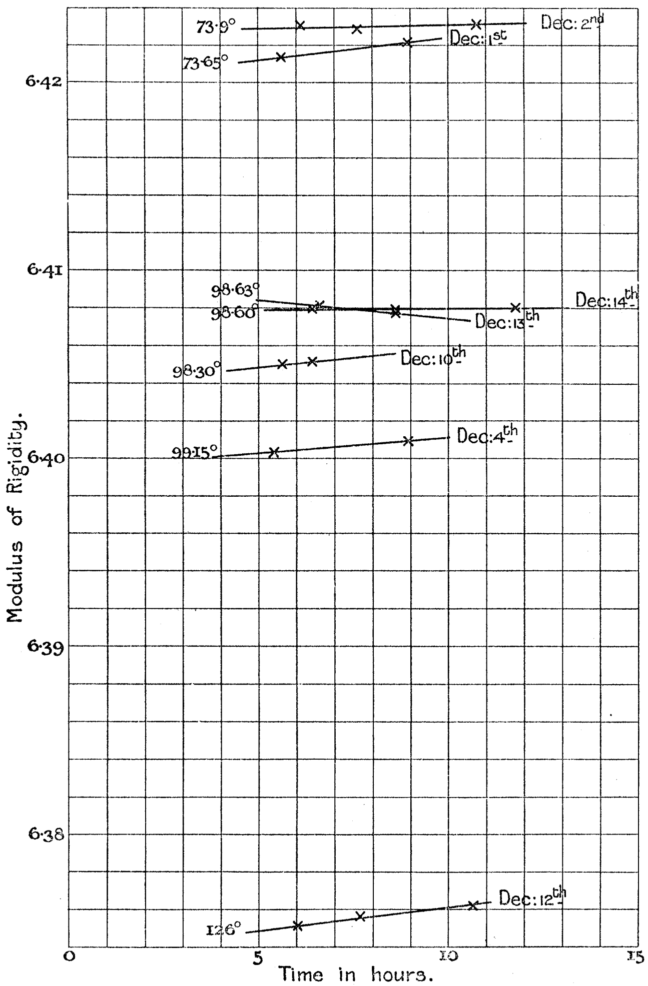

Diagram V. Showing the alteration of the modulus of rigidity of platinum wire with time.

It is curious that on December 13, the rigidity at 98·63° *decreased* as time went on. On the following day it was practically constant.

Table VI. contains the values of the logarithmic decrements obtained at the various temperatures. Each is the mean of two or three observations.

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TABLE VI.—Platinum Wire.

Date.	Temperature of wire.	Logarithmic decrement of oscillations.	Date.	Temperature of wire.	Logarithmic decrement of oscillations.
November 25 .	11·76	·000536	December 9 .	94·25	·000313
" 26 .	33·53	·000520	" 10 .	98·27	·000322
" 27 .	33·69	·000481	" 10 .	98·29	·000314
" 28 .	13·81	·000453	" 11 .	14·85	·000278
" 28 .	58·10	·000516	" 11 .	126·13	·000366
" 29 .	58·45	·000467	" 11 .	126·01	·000341
December 1 . .	16·34	·000398	" 12 .	125·93	·000329
" 1 . .	73·63	·000486	" 12 .	125·92	·000313
" 1 . .	73·64	·000451	" 13 .	16·91	·000246
" 2 . .	73·88	·000433	" 13 .	98·64	·000281
" 3 . .	12·25	·000306	" 14 .	98·64	·000281
" 3 . .	98·93	·000448	" 14 .	98·60	·000274
" 4 . .	99·07	·000414	" 14 .	98·55	·000274
" 5 . .	17·56	·000304	" 15 .	17·10	·000251
" 9 . .	16·25	·000294			

From the above table it will be noticed that, with the exception of the first heating, increase of temperature increases the logarithmic decrement. This result is the opposite to that obtained for soft iron. The logarithmic decrement was continually decreasing with the time, the rate of decrease being more rapid the higher the temperature. This time effect masks the effect of temperature to such an extent that no useful knowledge is obtained by plotting the results given in Table VI.

When the determinations of the rigidity had been completed, a series of observations was taken to see in what manner the period and logarithmic decrement varied with the amplitude of vibration of the vibrator. A set of observations was taken at temperatures very near to 19° C., and a second set at temperatures very near to 98·6° C. The observed periods were corrected to these temperatures.

The results of these experiments are recorded on Diagram VI., in which the continuous lines show the alteration of the period of vibration, and the dotted lines the alteration in the logarithmic decrement of the oscillations. It will be noticed that both the period and logarithmic decrement increase with the amplitude of vibration, and that the rate of increase of each is less at larger amplitudes than at smaller ones. The curves corresponding to the same temperature are of similar form.

Gold.

A pure gold wire was used. As with platinum and iron, the period of vibration diminished with time. The amount of this diminution got less and less on repeated heatings, and the period became finally nearly constant. The calculation of the rigidities was complicated by the fact that the wire gradually drew out under the

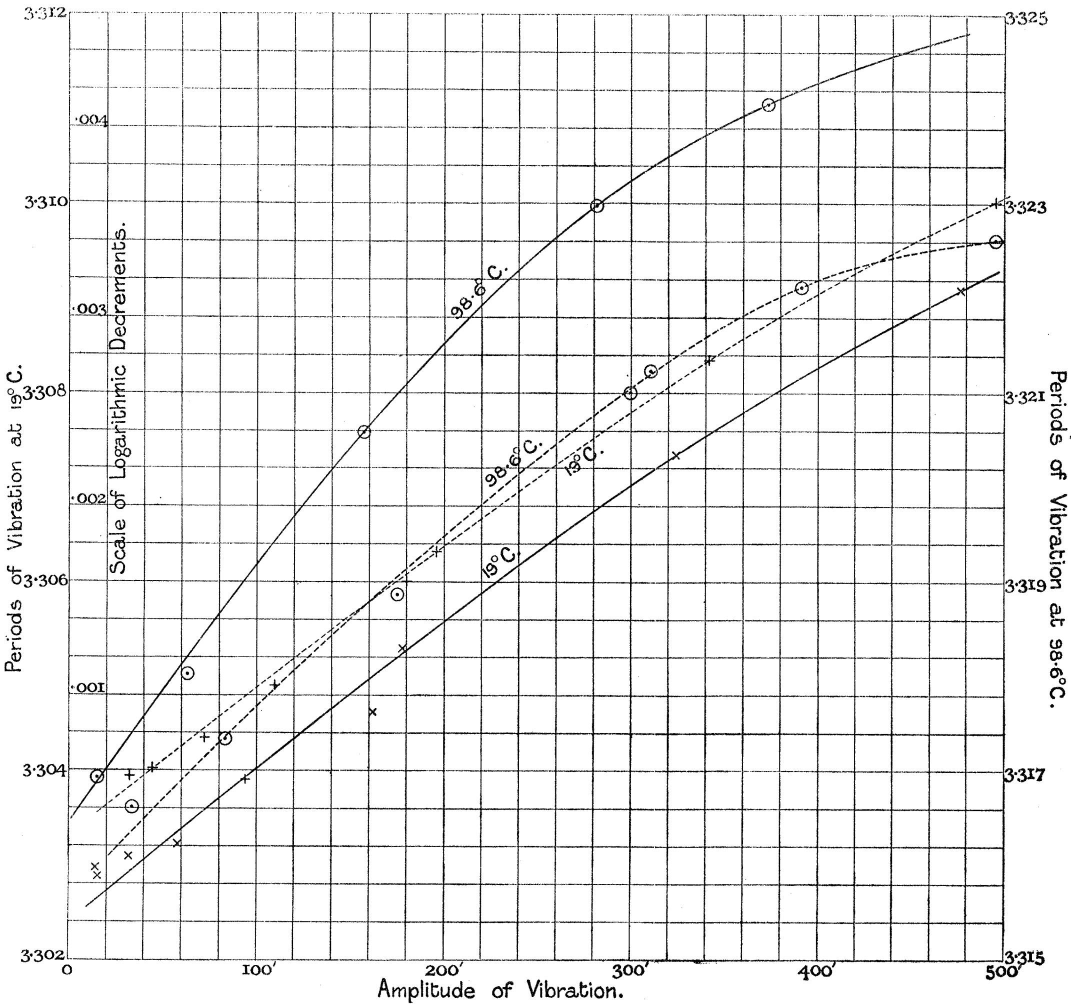

Diagram VI. Platinum wire. Dotted lines show the alteration of the logarithmic decrement of the torsional oscillations with the amplitude of vibrations. Continuous lines show the effect of the amplitude on the period of torsional vibration.

slight tension to which it was subjected. The increase of length amounted to ·1205 centim. at the end of the experiment.

The manner in which the rigidity modulus altered with the temperature can be seen from Diagram VII. The curve obtained is similar to that given by soft iron, but it is more nearly a straight line.

In order that the rate of increase of rigidity with time might be more easily seen, the rigidities at various temperatures were plotted against the lengths of time for which the wire had been kept at those temperatures. The resulting curves (which are not given) were similar to those obtained for iron and platinum.

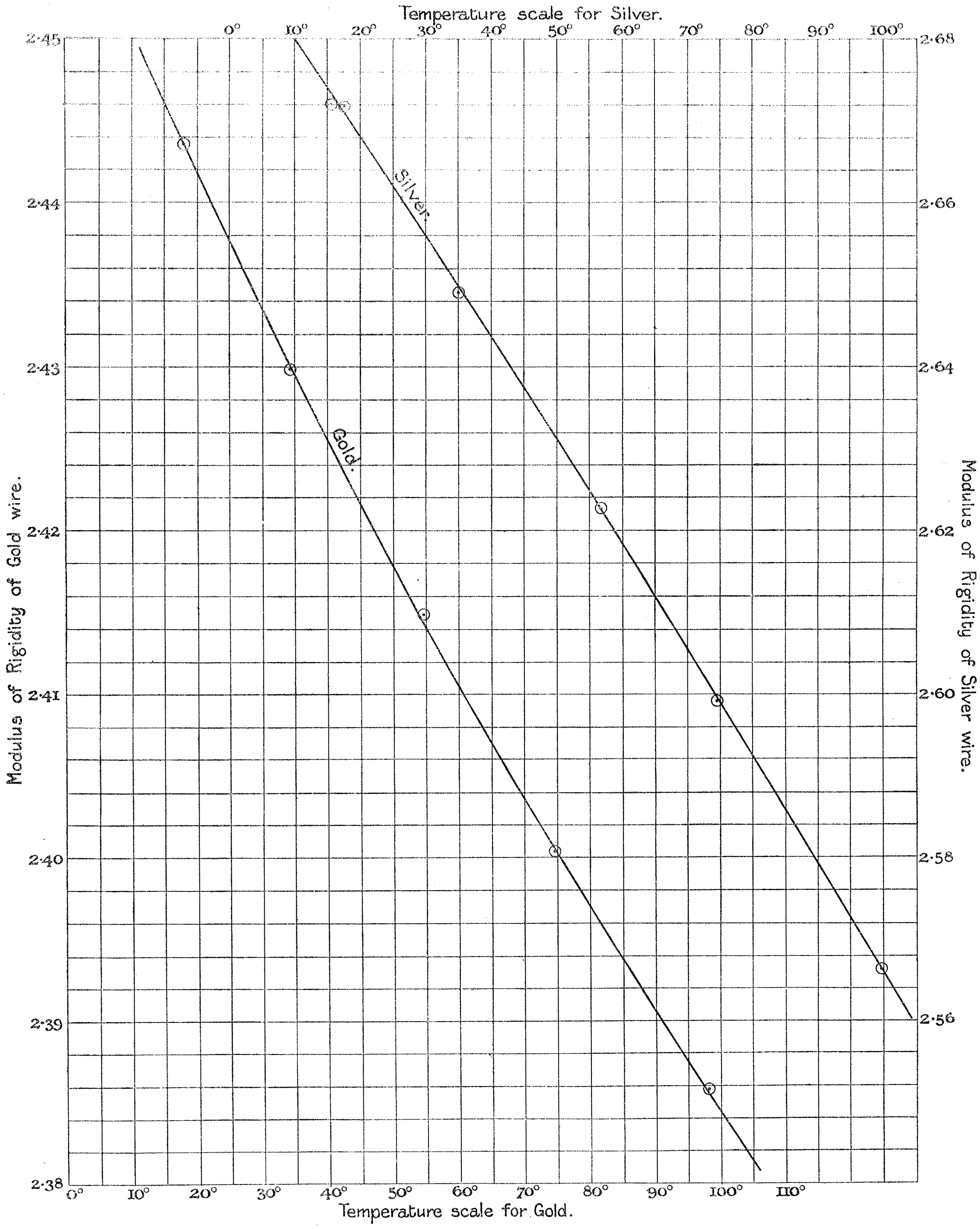

Diagram VII. Showing the effect of temperature on the modulus of rigidity of *gold* and of *silver* wire.

The value of the rigidity modulus at 15° C. for gold was found to be

$$n_{15} = 2{\cdot}4453 \times 10^{11} \text{ dynes per sq. centim.}$$

and the "temperature coefficient" $\beta = {\cdot}0003299$.

The curve obtained by plotting the values of the logarithmic decrements at the various temperatures was similar to that obtained for copper and showed that the logarithmic decrement increases as the temperature is raised, slowly at first, but more rapidly at higher temperatures. This was especially noticeable after passing 100° C.

Experiments were made to ascertain the manner in which the period and logarithmic decrement varied with the amplitude of vibration of the vibrator. Observations were taken at two temperatures, viz., 19·0° C. and 99·3° C. It was found that the logarithmic decrement curves for the two temperatures were very much alike, and showed that the logarithmic decrement increases with the amplitude of vibration, the rate of increase being slightly greater at the higher temperature. This is practically the same as the result obtained for platinum.

From the curves for the variation of period with the amplitude of vibration it was evident that the period increases with the amplitude, the rate of increase at the higher temperature being greater than at the lower.

Steel.

Two wires of pianoforte steel were experimented on, one in the first form of apparatus in which the vibrator was not enclosed in the heating jacket, and the other with the apparatus in its final form. Both were carefully annealed in the manner already described before the determinations of the rigidity were begun. With the first wire the method of experimenting was slightly different from that used with the other wires. The temperature in the heating jacket was regulated by means of a stream of water heated by passing through a gas furnace fitted with a quickly acting gas regulator. By this means the temperature could be kept constant for any length of time, and the effect of very small alterations of temperature observed. With this arrangement the temperature could not be raised above 36° C., but observations at 75° C. and 100° C. were also taken by employing the method of heating described earlier in this paper. I do not attach much importance to the numbers obtained, on account of the defective form of heating jacket employed, but the results can be relied on in so far as they show that the rigidity modulus of steel is a perfectly definite quantity, and that its rate of decrease with increasing temperature is constant also.

Table VII. contains the values of the rigidity modulus at the various temperatures. Each was calculated from the mean of a large number of observations (in most cases more than 20) at the temperature given. It must be stated that the results have been tabulated in descending order of temperature—not in the order in which they were obtained. This accounts for the fact that in two or three cases results are given at very nearly the same temperatures. These were taken at different times, weeks apart, in order to see if the rigidity came back to its original value after the wire had been heated to a higher temperature.

TABLE VII.—Steel Wire.

Temperature.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Temperature.	Modulus of rigidity in 10^{11} dynes per sq. centim.
99·35	8·2671	26·04	8·4265
76·41	8·3204	24·00	8·4316
35·04	8·4069	22·27	8·4357
34·33	8·4083	20·16	8·4404
34·31	8·4085	20·03	8·4406
32·78	8·4121	17·57	8·4461
31·25	8·4156	15·33	8·4504
28·97	8·4206	15·23	8·4508
28·50	8·4214	13·36	8·4553
26·04	8·4265	10·40	8·4613

On plotting the values of the modulus of rigidity against the corresponding temperatures a straight line was obtained. From this line the value of the modulus at 0° C. was found by exterpolation and then the true temperature coefficient α was determined. The value found was $\alpha = \cdot 0002543$, where $n_t = n_0(1 - \alpha t)$. The value of n at 15° C. was found to be $n_{15} = 8{\cdot}4517 \times 10^{11}$ dynes per sq. centim., from which, for the sake of comparison with the values for the other wires, the "coefficient" β was obtained, $\beta = \cdot 0002552$.

In determining the modulus of rigidity of the second steel wire with the later form of apparatus, use was made of the fact that it had already been found to be a linear function of the temperature. Observations were therefore taken at two temperatures only, and from these the temperature coefficient was determined.

The following are the means of several determinations at the temperatures given.

TABLE VIII.—Steel Wire.

Date.	Temperature.	Corrected period in seconds.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Time, in hours, during which temperature had been maintained.
May 26. . . .	17·10	2·914724	8·13254	—
" 27. . . .	99·22	2·946973	7·95552	4·00
" 27. . . .	99·20	2·946966	7·95556	8·25
" 28. . . .	18·63	2·915487	8·12829	—

From these the value of the modulus at 15° C. was found to be $n_{15} = 8{\cdot}13656 \times 10^{11}$ dynes per sq. centim., and the "coefficient" β to be, $\beta = \cdot 0002642$.

The results of the experiments by which the moment of inertia of the vibrator was found with this wire will be found in Part I. of this paper. Some experiments were made to determine the manner in which the period and logarithmic decrement varied with the amplitude of vibration of the vibrator. The results are given in Table IX.

TABLE IX.—Steel Wire.

Temperature.	Amplitude of vibration in minutes.	Logarithmic decrements.	Corrected period in seconds.
98·96	14·4	·0003186	2·946846
98·96	27·4	·0003180	2·946858
98·96	50·2	—	2·946862
98·96	121·3	·0003226	2·946861
98·96	205·0	·0003226	2·946875
19·40	15·2	·0003248	2·915714
19·40	28·9	·0003212	2·915784
19·40	59·3	·0003418	2·915724
19·40	127·3	·0003362	2·915728
19·40	237·4	·0003474	2·915721
19·40	15·2	·0003260	2·915709

From these it is seen that the logarithmic decrement of the vibrations is less at the higher temperature than at the lower one. This was also found to be the case with soft iron. It is also seen that the period increases only very slightly with the amplitude so long as this is small. The logarithmic decrements are slightly greater at the larger amplitudes, but the increase is not uniform.

Silver.

A pure silver wire was used, and it was found that the rigidity modulus for silver is much more nearly constant at a constant temperature than for most of the other metals examined. This was particularly evident in the case of the observations taken at the ordinary temperature, which were not very seriously altered by the successive heatings, especially in the case of those taken near the end of the series. There was, however, on the whole a slight increase of rigidity with time.

The rigidity-temperature curve for silver is given in Diagram VII. The points marked represent the first and second observations at the ordinary temperature of the room and the means of the observations at the higher temperatures. The line is slightly curved in the same direction as that for platinum (see Diagram IV.).

The value of the modulus of rigidity for silver at 15° C. was found to be $n_{15} = 2{\cdot}6741 \times 10^{11}$ dynes per sq. centim., and of the "temperature coefficient" $\beta = {\cdot}0004540$.

The logarithmic decrements of the amplitudes of vibration when plotted against the corresponding temperatures gave a curve similar to that obtained for copper (Diagram II.).

Observations on the effect of the amplitude of vibration on the period and logarithmic decrement were made, as usual, at two temperatures, 15°·00 C. and 99°·00 C.

It was found that—as with most of the other wires examined—both the period and logarithmic decrement increased with the amplitude of vibration, and that the rate of increase of each was greater at the higher temperature than at the lower one.

Oscillating the vibrator through a large amplitude was found to increase both the period and the logarithmic decrement at the smaller amplitude, indicating a diminution in the rigidity and an increase in the internal viscosity of the wire.

Aluminium.

The wire used was of pure aluminium, and the rigidity-temperature curve obtained was similar to that given by silver. Several peculiarities were, however, noticed with this wire. In the first place, heating to 59° C. diminished the value of the rigidity modulus at 17° C., and this diminution was much larger after the wire had been heated to 75° C., but after heating to 100° C. the rigidity at the laboratory temperature was increased, and this increase was continued after each heating to 100° C. Successive heatings to 100° C. sometimes increased and sometimes diminished the value of the rigidity modulus at that temperature. It is also curious that, when kept at a constant temperature of 59° C. or 74° C., the rigidity diminished slightly as time went on, but that at 100° C. it gradually increased with time. It thus appears that the effect of heat on the rigidity of an aluminium wire is very irregular.

From the curve obtained, the modulus of rigidity for aluminium at 15° C. was found to be $n_{15} = 2{\cdot}5478 \times 10^{11}$ dynes per sq. centim., and the value of the "temperature coefficient" $\beta = {\cdot}001351$.

The logarithmic decrements of the amplitudes of the torsional vibrations were found to increase with the temperature, but the value at any one temperature decreased as time went on. On plotting the logarithmic decrements against the corresponding temperatures, the curve drawn through the mean of the observations at each temperature was similar to that obtained for copper.

Observations made at larger amplitudes of vibration showed that both the period and logarithmic decrement increased with the amplitude, the increase being more rapid at 98·00° C. than at 19·50° C. The effect of swinging through a large amplitude (5°) was to lessen the value of the logarithmic decrement, and to increase the period of vibration at smaller amplitudes.

Tin.

A pure tin wire was used. Owing to the great internal viscosity of this metal, the torsional periods could not be timed with the accuracy obtained in the case of the harder metals; for the oscillations damped down so quickly that only a comparatively small number took place. The difficulties of observation were also increased by the fact that, although a light aluminium vibrator was used with the wire, it underwent a considerable elongation, especially at the higher temperatures. At the second heating to 100° C. this elongation was so great that the vibrator moved about in jerks as the wire gave way, and rendered timing impossible.

It was found that the rigidity of the wire at the ordinary laboratory temperature was diminished by repeated heatings, and that when the wire was allowed to rest at one temperature its rigidity increased as time went on. The curve in Diagram VIII. is drawn through the mean of the observations at the different temperatures.

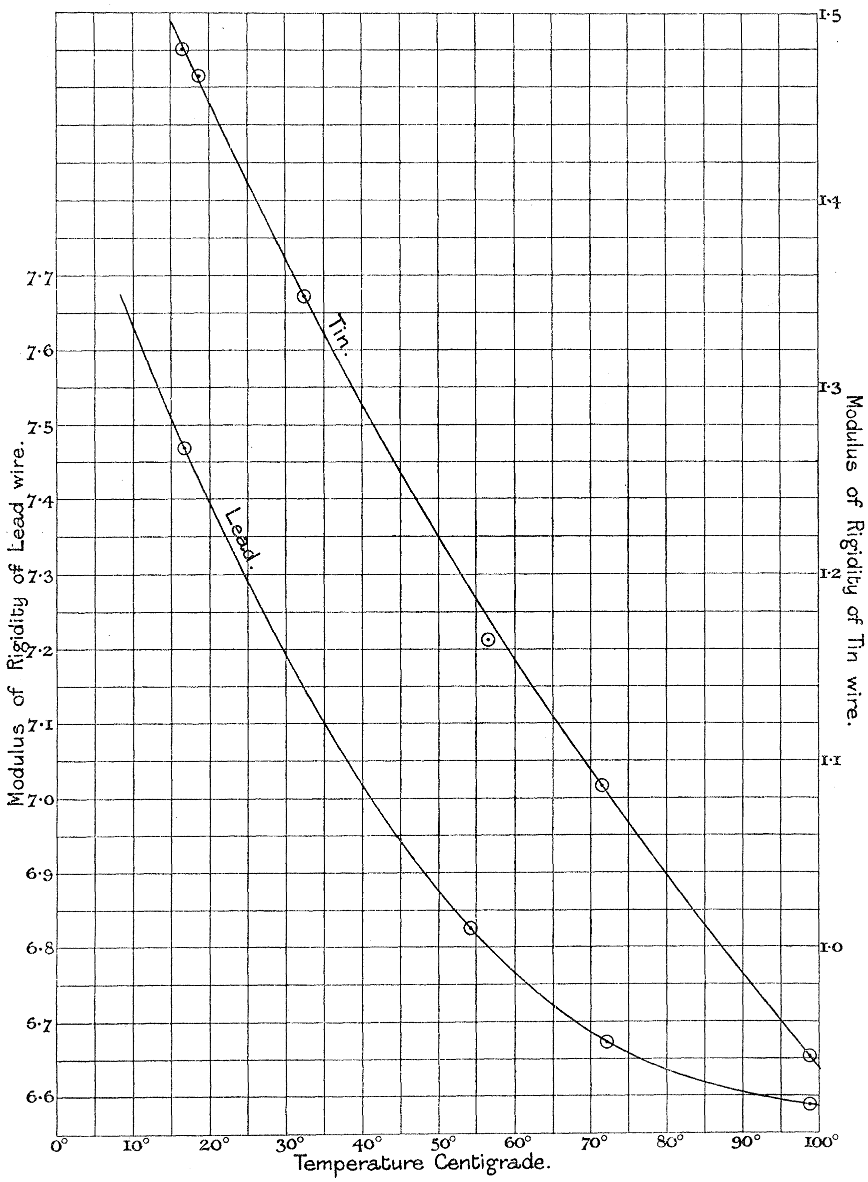

Diagram VIII. Showing the effect of temperature on the modulus of rigidity of tin and of lead wire.

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The value of the rigidity modulus at 15° C. was found to be $n_{15} = 1{\cdot}4960 \times 10^{11}$ dynes per sq. centim. and the "temperature coefficient" $\beta = {\cdot}005942$.

Some experiments were also made with another pure tin wire which had not been previously annealed. The range of temperature employed in this case was only 34° C. The value found for the rigidity modulus at 15° C. was $n_{15} = 1{\cdot}8715 \times 10^{11}$ dynes per sq. centim. and for the "temperature coefficient" $\beta = {\cdot}00385$. The rigidity of tin is thus lessened by annealing. The "temperature coefficient" is much the smaller in the case of the unannealed wire, but the alteration in the rigidity modulus per 1° rise of temperature is more nearly equal in the two cases.

The values of the logarithmic decrements of the amplitudes of oscillation obtained during the experiments with the annealed wire showed an increase in the logarithmic decrement with the temperature, and also a continual diminution with time.

Observations of the logarithmic decrement of the oscillations of the unannealed tin wire were made, and the results showed that the rate of increase of the logarithmic decrement with temperature was much greater than in the case of the annealed wire. The heating of the unannealed wire greatly reduced the value of the logarithmic decrement at the ordinary laboratory temperature.

Some observations on the effect of increased amplitude of vibration on the logarithmic decrement gave very irregular results, complicated by the fact that the logarithmic decrement of a constant amplitude diminished as time went on. The observations showed that the logarithmic decrement increased with the amplitude of oscillation, but in a very irregular manner.

Lead.

The wires used were drawn down from a strip cut from a sheet of lead. In order to try to avoid the complications caused by the wire elongating under the slight tension to which it has to be subjected, series of observations at different temperatures were taken all on the same day.

Observations were first taken with an unannealed lead wire, and it was found that the rigidity increased very considerably as time went on, especially at the higher temperatures. The rigidity-temperature curves obtained on different days were very similar, indicating that the alteration of rigidity with temperature was practically constant, although the absolute value increased as time went on. A specimen of the curves obtained is given in Diagram VIII.

The value of the rigidity modulus at 15° C. is $n_{15} = 7{\cdot}505 \times 10^{10}$ dynes per sq. centim. The mean value of the decrease per 1° rise of temperature at 15° C. was ${\cdot}0225 \times 10^{10}$ dynes per sq. centim., from which the "temperature coefficient" $\beta = {\cdot}00300$.

The logarithmic decrement of the amplitude of the torsional oscillations increased with the temperature, but this effect was complicated by a very considerable decrease in the value with time. The results, however, showed that the rate of increase was much greater at higher temperatures than at the ordinary temperature of the laboratory.

Some observations were made at the temperature of the laboratory and at 100° C., on a lead wire which had been carefully annealed. It was found that the rigidity increased only very slowly with the time, and that the increase per 1° C. over this range of temperature was nearly the same as with the first wire. This might perhaps have been expected, for in the final values of the modulus of the first wire a certain amount of annealing must have taken place owing to the wire having been kept for some time at 100° C. The increase of the modulus per 1° C. was for the annealed wire $\cdot 0130 \times 10^{10}$ dynes per sq. centim., and for the unannealed wire $\cdot 0127 \times 10^{10}$ dynes per sq. centim. The value of the modulus at 15° C. for the annealed wire was $n_{15} = 7\cdot 977 \times 10^{10}$ dynes per sq. centim.

Observations of the logarithmic decrement of the amplitudes of the torsional oscillations of this wire gave results similar to those obtained with the unannealed wire.

Cadmium.

The wire was of pure cadmium, and was first annealed in the manner already described. The small rectangular brass vibrator was used, but its weight caused the wire to elongate very considerably at the higher temperatures, and the results obtained at those temperatures were therefore unreliable. A new wire was then used, and the temperature coefficient determined from observations at the temperature of the laboratory and at 32° C., the experiments being done as quickly as possible in order that the radius of the wire might be determined before it had drawn out to any serious extent. The total elongation was ·052 centim., and a correction on this account was applied in determining the rigidity.

The value of the rigidity modulus at 15° C. was found to be $n_{15} = 2\cdot 3124 \times 10^{11}$ dynes per sq. centim., and the "temperature coefficient" ·005855.

The logarithmic decrement of the amplitude of the torsional oscillations increased with the temperature, the rate of increase being much greater at 100° C. than at the ordinary laboratory temperature.

Commercial Copper.

An ordinary copper wire of the same size as the other wires used was carefully annealed in an atmosphere of carbonic acid gas, and a short series of observations taken with it, the object being to ascertain if there was any striking difference between the behaviour of this wire and that of the pure copper wire already described.

It was found that the rigidity at a constant temperature was far from being constant, in marked contrast to what was obtained with the pure copper wire. On plotting a series of values of the modulus of rigidity against the corresponding temperatures a curve, and not a straight line, was obtained. The form of the curve was similar to that given by gold, iron, &c., and suggests that the departure from the linear law is due to the gradual increase of rigidity with time.

The value of the modulus at 15° C. for commercial copper is $n_{15} = 3{\cdot}7956 \times 10^{11}$ dynes per sq. centim., and the "temperature coefficient" $\beta = {\cdot}0004074$.

The modulus of rigidity is thus considerably less, and the temperature coefficient rather greater, than in the case of pure copper.

The logarithmic decrements of the amplitudes of oscillation were about four times as large as those obtained with the pure copper wire, indicating a much greater internal viscosity in the case of this commercial copper.

PART III.

Summary and Comparison of Results.

The results of the experiments may be summarised as follows:—

1. In all the materials examined, with the exception of pure copper and of steel, the modulus of rigidity at one temperature is not constant, but increases as time goes on. The rate of increase of rigidity with time is greater the higher the temperature, and repeated heatings to the same temperature gradually lessen the rate of alteration with time at that temperature, but even in the course of months of experimenting the increase of rigidity with time cannot be entirely eliminated.

2. The diminution of the modulus of rigidity per degree rise of temperature between 10° C. and 100° C. is constant for pure copper and for steel, but not for any of the other materials examined.

3. In the case of the metals iron, gold, tin, lead, and commercial copper, the rigidity-temperature curve is of such shape as to suggest that if it were possible to obtain the values of the rigidity modulus at the different temperatures, all within a very short space of time (so as to avoid the "time effect"), the resulting curve would be a straight line.

4. In the case of the metals platinum, silver, and aluminium, the shape of the rigidity-temperature curve shows that the effect of the gradual increase of rigidity with time is such as to make the alteration with temperature approximate more closely to a linear law than would be the case if the observations were all taken within a very small interval of time. For these metals, therefore, the decrease of torsional elasticity per 1° C. rise of temperature increases with the temperature.

5. In general, the effect of heating to a high temperature is to increase the value of the rigidity modulus at low temperatures. [This applies even to pure copper, of which the modulus of rigidity at the ordinary laboratory temperature is slightly greater after the wire has been heated to higher temperatures. The rigidity of steel

was quite constant, and with silver the value of the modulus at the temperature of the laboratory, in the last few experiments, was unaltered by a temporary increase of temperature. The tin wire was the only case in which the rigidity modulus at the ordinary laboratory temperature was lessened by heating.]

6. The internal viscosity of all the metals examined, with the exceptions of soft iron and steel, increases with the temperature. This increase varies very much with different metals, being greatest with aluminium and least with platinum. The internal viscosity of soft iron decreases rapidly with rise of temperature, and reaches a minimum value at about 100° C. There is a slight decrease also in the case of steel.

7. Repeated heating and continual oscillation through small amplitudes decrease the internal friction.

8. Both the internal friction and the period of torsional vibration increase with the amplitude of oscillation. The increase is generally greater the higher the temperature of the wire. It is least in the case of steel and is small in the case of soft iron.

9. Vibration through a large amplitude considerably alters both the logarithmic decrement and period of oscillation at smaller amplitudes. The nature of the alteration varies with different metals, being in some cases an increase and in some a decrease.

10. The internal viscosity of a well annealed wire suspended and left to itself gradually decreases.

11. The internal viscosity of an unannealed wire is enormously reduced by annealing.

The values of the modulus of torsional rigidity obtained in the present research are collected together in Table X., which also contains the values given by the observers mentioned in Part I. of this paper, who have investigated the variations of the modulus with changes of temperature.

TABLE X.

Observer.	Modulus of rigidity at 15° C. in 10^{11} dynes per sq. centim.										
	Copper.	Commercial copper.	Iron.	Steel.	Platinum.	Gold.	Silver.	Aluminium.	Tin.	Lead.	Cadmium.
KUPFFER	—	—	7·137	—	6·249	2·658	2·775	—	—	—	—
KOHLRAUSCH and LOOMIS	3·824	—	6·808	—	—	—	—	—	—	—	—
PISATI	3·882	—	7·930	8·112	6·030	—	2·503	—	—	—	—
TOMLINSON	4·219	4·487	7·585	—	6·795	—	2·667	2·601	1·186	·726	—
KATZENELSOHN	3·520	—	7·363	—	7·271	3·875	2·503	3·286	—	—	—
SUTHERLAND	—	—	—	—	—	—	—	—	1·54	·882	—
GRAY, BLYTH and DUNLOP	4·2596	4·4822	8·2855	7·9648	—	—	—	—	—	—	—
SCHÆFER	3·886	—	7·19	—	6·46	—	2·420	2·284	—	·539	1·004
HORTON	4·3693	3·7956	8·2620	8·4517	6·4600	2·4453	2·6741	2·5478	1·4960	·7977	2·3124
—	—	—	7·9656	8·1366	—	—	—	—	—	—	—

The numbers in the above table show considerable discrepancies in the value found for the rigidity modulus of any one substance by different observers. In many cases this is no doubt due to want of purity and to imperfect annealing of the specimens used, also perhaps in some cases to the fact, that the wire was subjected to excessive tension during the rigidity determinations.

Owing to the fact that different observers have expressed the variation of rigidity with temperature by different formulæ (already quoted in Part I. of this paper), it is not easy to give a comparison of their results. It appeared to be best to calculate in each case the average decrease in rigidity per 1° C. rise of temperature per unit value of the modulus over the temperature 0° C. to 100° C., this being the most generally employed range of temperatures. In order to obtain numbers from the present research to compare with those of other observers, a temperature coefficient was calculated for each wire from the last set of observations at 100° C., and at the ordinary laboratory temperature, assuming the rate of alteration of rigidity with temperature to be constant in the interval. These numbers are thus the values of α in the equation $n_t = n_{15}\,[1 - \alpha\,(t - 15)]$. The coefficient β—the alteration per 1° C. rise of temperature per unit value of the rigidity modulus, at 15° C.—are also included in Table XI.

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TABLE XI.

Observer.	Coefficient of diminution of rigidity per unit per °C.										
	Copper.	Commercial copper.	Iron.	Steel.	Platinum.	Gold.	Silver.	Aluminium.	Tin.	Lead.	Cadmium.
KUPFFER	—	·0006907	·0004119	·0004708	·0002201	—	·0005840	—	—	—	—
NAPIERSKY	—	—	·000400	—	—	—	·000345	—	—	—	—
KOHLRAUSCH and LOOMIS	·000572	—	·000483	—	—	—	—	—	—	—	—
PISATI	·000296	—	·000215	·000237	·000127	—	·000437	—	—	—	—
TOMLINSON	·0002921	—	·0002693	—	·0000744	—	·0003938	·0005724	—	—	—
KATZENELSOHN	·000365	—	·000310	—	·000164	·000285	·000723	·00213	—	·00800	—
SUTHERLAND	—	—	—	—	—	—	—	—	·00362	·00227	—
GRAY, BLYTH, and DUNLOP	·00111	·000392	·00041	·000338	—	—	—	—	—	—	—
SCHÆFER	·000449	—	·0003035	—	·000178	·000301	·000821	·00247	·00820	·00787	·00467
BENTON	·000442	—	—	·000476	—	—	—	—	—	—	—
HORTON	·0003877	·0003795	·0002884	·0002642	·0001196	·0003069	·0004784	·001479	·004160	·001640	—
,, (values at 15° C.)	·0003877	·0004074	·0007339	·0002642	·0001018	·0003299	·0004540	·001351	·005942	·00300	·005855

The values given in the above table were not in all cases found from the range of temperatures 0° C. to 100° C. The experiments of NAPIERSKY and those of KUPFFER on iron, platinum, and silver, were only performed over a small range of temperature, from about 5° C. to 25° C., while the numbers of SCHÆFER and BENTON apply to temperatures between − 186° C. and + 20° C. It must also be pointed out that KOHLRAUSCH and LOOMIS, PISATI and TOMLINSON, give formulæ showing that the alteration of rigidity with temperature does not follow a linear law; KOHLRAUSCH and LOOMIS, and TOMLINSON, finding that the rate of decrease per degree rise of temperature increases with the temperature, a view with which PISATI'S results, with a few exceptions, agree. The values given by KOHLRAUSCH and LOOMIS are very different from those obtained by other observers, a result no doubt due in part to the fact that the wires they used had not been subjected to any annealing process. These authors state that observations taken on different days could not be compared, and from their tables it is seen that the period of torsional oscillation decreased from day to day. This means an increase of rigidity with time, an effect to which I have drawn special attention in this paper, and which, in unannealed wires, is quite large.

A fault common to all the experiments which have hitherto been made by the dynamical method, is that sufficient precautions were not taken to insure that during the observations at high temperatures the wire was everywhere throughout its length at the same temperature. The average temperature of the wire must generally have been considerably below the temperature indicated by the thermometers. This was found to be the case when experimenting with the first form of apparatus used in the present research. Under these circumstances, the observed rate of alteration of rigidity with temperature would be less than the actual alteration, and the temperature coefficients obtained would be too small. In the present research the values of the temperature coefficients for iron and steel, as given by experiments with the old form of apparatus, are less than the values obtained with the apparatus in its final form.

The results of KUPFFER and of NAPIERSKY are only of interest from an historical point of view, they differ very considerably from those obtained by more recent observers. One great fault in their experiments is that the amplitudes of oscillation were extremely large. In PISATI'S experiments, too, the amplitude of vibration was 90° at the beginning of each observation, and the rigidity of the wire (the length of which was 65 centims.) must have been considerably altered by being subjected to this treatment.

Of the more recent experiments, those of SCHÆFER stand out as comprising the largest number of materials. As already stated, he employed the statical method of experimenting, observing the torsional deflections of a wire under a constant couple. The temperatures worked at were, in general, only two, the temperature of the laboratory and that of boiling liquid air, and the rate of alteration of rigidity with temperature was assumed to be constant between them. The main object of this work was, however, to compare the temperature coefficient of the torsion modulus with the coefficient of expansion, melting-point, and atomic weight of the metal in question, rather than to obtain great accuracy in the investigation of the effects of alteration of temperature on the rigidity modulus.

Although observations of the logarithmic decrement of the amplitude of the torsional oscillations were not the main object of this research, the results obtained throw considerable light on the internal viscosity of the wires used, and it seems desirable, therefore, to compare them with similar observations by other observers.

The diminution of amplitude of the torsional vibrations of a wire suspended as in the present experiments is due to two causes :—

(1) loss of energy by the vibrating system to the surrounding air ;

(2) the internal molecular friction of the suspended wire.

In order, therefore, to obtain accurate information as to the internal friction of the wire, it is necessary to deduct from the observed logarithmic decrement that portion which is due to the viscosity of the air. An exact calculation of this is impossible in the present case on account of the various objects fixed inside the box enclosing the vibrator, but an approximate value was found by using MAXWELL's formula for the case of a disc vibrating between two fixed plates. The value thus found for the logarithmic decrement due to air-damping was of the order 10^{-5}. The increase in this value due to an increase of the temperature of the air of 100° C. (about 25 per cent.) would not be within the limits of accuracy of the observations, and hence the variation in the observed logarithmic decrement on increasing the temperature of the jacket enclosing the vibrating system may be taken as being due entirely to the alteration of the internal viscosity of the suspending wire.

Observations of the effects of changes of temperature on the internal viscosity of metal wires have been made by most of the observers who have investigated the modulus of rigidity by the dynamical method, notably by KUPFFER, PISATI, and TOMLINSON. KUPFFER was probably the first to observe that the rate of subsidence of torsional oscillations increased with the temperature of the wire. He did not investigate this effect very fully, merely stating that at each temperature the amplitudes of successive oscillations decrease in geometrical proportion, a conclusion which is not borne out by the numbers given. PISATI, who experimented over a range of temperatures from 0° C. to 300° C., investigated this effect more fully, and gives a series of formulæ for calculating the amplitude of the n^{th} oscillation after the amplitude was 90°, over different ranges of temperatures. He found a marked increase in the rate of subsidence at higher temperatures except in the case of iron, in which the number of oscillations made by the wire in damping down from an amplitude of 90° to an amplitude of 10° at first *increased* with rise of temperature and reached a maximum at 100° C., afterwards decreasing again and becoming constant between 200° C. and 300° C. This minimum internal viscosity for iron at

100° C. was also observed in the present research, but it is not mentioned by many experimenters.

STREINTZ* was the first to lay stress on the fact that there is a tendency in metallic wires to accommodate themselves to a definite elastic state. In wires of brass, silver, platinum, copper, iron, and steel examined by him there was a marked diminution in the rate of subsidence produced by keeping the wire for long periods in continual torsional oscillation. Lord KELVIN† had previously stated that the rates of subsidence of the vibrations of several wires tested by his students were much less rapid after they had been allowed to rest for a week-end than when kept in continual vibration. The present research shows that the whole question depends on the amplitude of the vibration. It is possible that the wires mentioned by Lord KELVIN (which had been kept "as continually as possible in a state of vibration," presumably by impulses administered by students' hands) had been subjected to larger amplitudes of oscillation than were used in the case of the wires which were allowed to rest between the successive observations. In my own experiments on the effect of the amplitude of vibration, in which the greatest amplitude used was about 8° or 9°, it is shown that, in general, vibration through a large amplitude very considerably alters both the period and logarithmic decrement of the oscillations at smaller amplitudes. Recently THOMPSON‡ has investigated this question and arrived at the conclusion that, when the temperature and amplitude of vibration are constant, both the period and logarithmic decrement are constant quantities, but his experiments do not seem to be capable of sufficient accuracy to establish the point.

STREINTZ expressed the results of his experiments in the following three laws, the first of which is not strictly borne out by his own results :—

a. The logarithmic decrement is independent of the amplitude of vibration, of the tension in the wire, of the period of oscillation, of the length and perhaps also of the diameter of the wire.

b. The logarithmic decrement varies with the chemical and physical nature of the wire. It increases with rise of temperature and is less in annealed than in unannealed wires.

c. The logarithmic decrement tends—up to a certain limit—to diminish with the number of oscillations performed by the wire.

Perhaps the most exhaustive experiments on the internal viscosity of wires have been made by TOMLINSON.§ He obtained a decrease of the logarithmic decrement in the case of iron, nickel, and platinum, on raising the temperature from 0° C. to 100° C., the percentage decrease being much the greatest in the case of iron. He states that

* STREINTZ, 'Sitzb. d. Wien. Akad.,' LXIX., Abth. 2, p. 337, 1874; also LXXX., Abth. 2, p. 397, 1880.

† THOMSON, 'Proc. Roy. Soc.,' vol. 14, p. 289, 1865.

‡ THOMPSON, 'Phys. Rev.,' VIII., p. 141, 1899.

§ TOMLINSON, 'Phil. Trans.,' vol. 177, p. 801, 1886; also 'Proc. Roy. Soc.,' vol. 40, p. 343, 1886.

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"if we start with a sufficiently low temperature the internal friction of all annealed metals is first temporarily decreased by rise of temperature and afterwards increased. The temperature of minimum internal friction is for most annealed metals between 0° C. and 100° C." In the present research this effect was obtained in two cases only, viz., for soft iron and for steel. There was, however, a slight decrease in the value of the logarithmic decrement at the first rise of temperature in the case of platinum and of commercial copper, but this was, I think, due to the "time effect" (a decrease of internal viscosity with time) being greater than the increase due to the rise of temperature. This time effect diminishes as time goes on and the decrease of logarithmic decrement with increasing temperature did not occur again. TOMLINSON'S other results are practically the same as those of STREINTZ, mentioned above. He, too, states that the logarithmic decrement of the torsional oscillations is independent of the amplitude of vibration, provided the deformations produced do not exceed a certain limit, varying with the nature of the metal. This is contrary to the results obtained in the experiments now described, in which it was found that the logarithmic decrement increased as the amplitude was increased from 14′ to 9°. In some cases the rate of increase was greatest at the smallest amplitudes, and it may be that in these cases, over larger amplitudes, the logarithmic decrement remains constant, but this is not a general law.

TOMLINSON also states that the logarithmic decrement of the amplitudes increases with the period of vibration, on this point differing from STREINTZ. In the present research it was found that the period increases with the amplitude of vibration. If, then, TOMLINSON is right, it would be expected that the logarithmic decrement would also increase with the amplitude of vibration, and not, as he states, be independent of it.

PART IV.

THE DETERMINATION OF COEFFICIENTS OF LINEAR EXPANSION.

The coefficients of expansion of the metal wires, and also of the gun-metal bar cast at the same time as the ring used in determining the moment of inertia of the vibrator, were determined by means of the measuring bench in the Physical Laboratory of the University of Birmingham. This excellent instrument was designed by Professor POYNTING, and he very kindly allowed me to work with it during the Cambridge vacations. As no description of this measuring bench has yet been published, I will describe it in detail, as well as giving the special arrangements adopted for measuring the expansion of wires.

A diagram of the apparatus is given in fig. 4. It consists of two microscopes held rigidly in vertical positions by massive iron stands, which rest on a slate window-slab and can be placed at any required distance apart. The microscopes project over a railway, along which two troughs, one containing the article under test and the other a standard metre, can be moved. The microscopes M, M (fig. 4) are each fitted with a rocking-plate micrometer,* by means of which measurements to ·0001 centim. can be made.

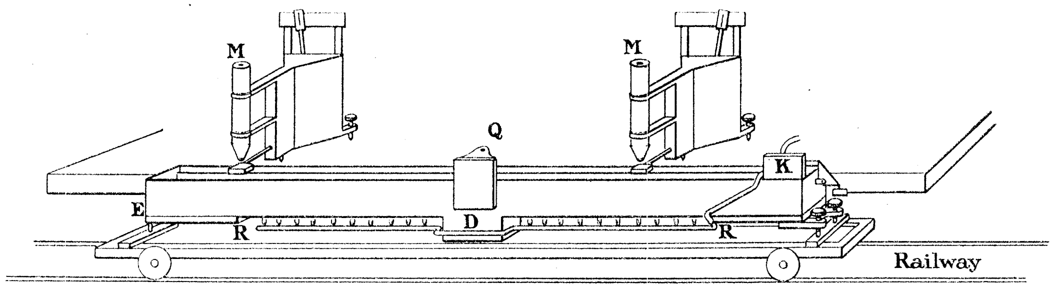

Fig. 4.

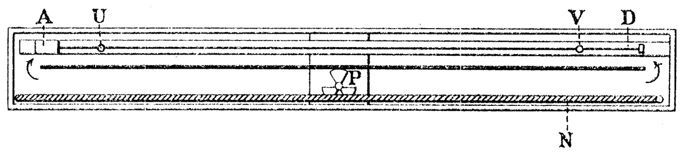

Fig. .

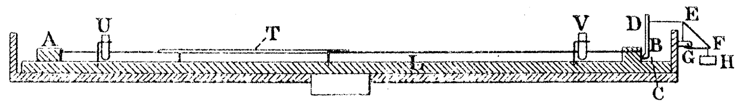

Fig. 6.

The trough in which the article to be measured is placed is a copper-lined box about 120 centims. long by 18 centims. high and 10 centims. deep. It is divided into two halves by a partition down the middle, stopping short at about 5 centims. from each end. This serves to direct the circulation of the water with which the trough is filled. The article under test lies on the side of this partition nearer to the slab on which the microscopes are supported. The arrangement may be seen in plan in fig. 5. The water is stirred by means of a fan P (fig. 5) worked by a motor Q (fig. 4). The fan revolves in a box forming a depression in the floor of the trough at its middle. It is situated on the opposite side of the central partition to that occupied by the article whose expansion is to be determined, and drives the water up on that side, thus causing a circulation in the direction shown by the arrows in fig. 5.

The trough containing the standard metre is exactly similar to the one just described. The metre is supported in a horizontal position on four rollers 25 centims. apart. The first and second, and the third and fourth of these are connected together by light levers having fulcrums at their centres, which are supported from the bottom of the trough. This method allows of the metre being supported without bending and also leaves it quite free to expand.

* For description, see POYNTING, 'Phil. Trans.,' A, vol. 182, p. 589, 1891, or "The Mean Density of the Earth," p. 95.

The arrangements for holding the wire are best seen in fig. 6. One end, A, is firmly clamped and the other is attached at B to a vertical lever CBD, of which the fulcrum is at C. A string is attached to the lever at D, and this passes down the grooved side of the metal piece EFG (which serves the same purpose as a frictionless pulley-wheel) and carries at its lower end the mass H. This mass is so chosen that the tension produced in the wire is the same as that due to the weight of the vibrator in the rigidity experiments.

The wire is surrounded by water except at the points the distance apart of which is to be measured. At these points it passes transversely through small brass tubes closed at the lower ends. These tubes (U and V, fig. 6) are supported in a vertical position by wires from the base bar L. The holes in the tubes were made the same size as the cross-section of the wire and just outside, both wire and tube were covered with melted shellac, which effectually prevented any water leaking in. The standard thermometer T was supported beside the wire by means of supports from the base plate L.

The water in the trough can be heated by means of the row of burners RR, shown in fig. 4. The gas feeding these passes through a gas regulator K, the thermal part of which is a mercury bulb, 1·3 centims. in diameter, running the whole length of the trough on the opposite side of the central partition to the wire. With this heating and stirring arrangement the water in the trough can be kept at a temperature not varying by more than ·05° C. for hours.

The top of the trough was covered over with pieces of bright tin plate, a small circular hole being left under each of the microscopes. These holes were also covered over except when readings were being taken. The thermometer was read to ·01° C. by means of a microscope, the covering of the trough above the part of the stem which was required to be seen being removed for this purpose. In order that the rocking plates and objectives of the microscopes might not be clouded with steam at the higher temperatures, a layer of ordinary lubricating oil was poured over the surface of the water. This effectually prevented any steam escaping, and also helped in maintaining the temperature constant. The microscopes were arranged on the slab so that each was over the corresponding tube on the wire, and particular care was taken to see that the wire was horizontal and parallel to the direction of motion of the trough. The height of the second trough containing the standard metre was adjusted so that the scale could be seen clearly without re-focusing the microscopes.

The difficulty in measuring small alterations in length directly by means of a microscope is to obtain a sufficiently fine fiducial mark. This difficulty is especially great in the case of a wire, on which it is very difficult to make a fine mark. After many failures it was found that the best plan was to map out the appearance of the surface of the wire and fix on some easily distinguishable point. The illumination was secured by an excellent arrangement designed by Professor POYNTING. By means of this the light from a small 10-volt electric lamp, enclosed at the end of a horizontal metal tube some 20 centims. long, can be focussed on to the wire, being reflected downwards by a right-angled prism. The path of the reflected beam is almost vertical and is in a plane perpendicular to the wire. In this manner perfect illumination is secured, and when looking at the standard metre no shadows are seen along the graduation lines.

In determining the coefficient of expansion of the gunmetal bar it was supported in a similar manner to the standard metre. The fiducial marks were near the ends of the bar and were made by brightly polishing the surface and then drawing a fine needle across. The marks were protected from the water by two short glass tubes which had been ground level at the bottom and were held firmly against the bar by stout elastic bands, a ring of thin sheet indiarubber being placed between.

In experimenting, the trough containing the wire under test was left under the microscopes only while readings were being taken. It was moved off along the line to be heated up. At each temperature about half-an-hour was allowed after the required temperature had been obtained before the readings were taken. During this period the heating and stirring were continual. The trough was then wheeled under the microscopes and adjusted in position, so that the marks were approximately under the cross wires. The heating and stirring were then stopped—for the shaking rendered the image indistinct—and by means of the rocking plates the marks were adjusted on to the cross wires. The temperature was then noticed and the trough wheeled away. The readings of the micrometer scales were then taken. This was repeated three or four times at each temperature, and in general there was no difficulty in obtaining a very close agreement.

The expansions were taken at various temperatures up to 90° C., and after each observation the distance between the microscopes was checked by wheeling the standard metre beneath them. This precaution was necessary in order to see that the heat did not warp the stands and thus alter the positions of the microscopes, but only in a few cases was it necessary to apply a correction on this account.

The coefficients of expansion of all the wires used in the rigidity experiments were determined, with the exception of the very soft metals lead, cadmium, and tin, which were found to draw out slightly under the necessary tension, especially at the higher temperatures. The coefficients of expansion of these metals would have to be found from bars of the material, but in the present research it was not necessary to know them with great accuracy on account of the fact that the rigidity determinations in these cases were not so accurate as with the harder wires. The values given in LUPTON's tables (1901) were therefore used.

Before determining the coefficients of expansion, the wires—which had been bent into spirals for weighing in air and water in the determination of their radii—were straightened and annealed in the same manner as before the determination of their rigidities.

The lengths of the wires at the different temperatures were plotted against the corresponding temperatures, and in every case the resulting curve was a straight line. The lengths of the wires at 0° C. were found by exterpolation and the values of the coefficients of expansion calculated. In order to show the accuracy obtained, the lengths of the wires at the different temperatures were calculated from the formula $l_t = l_0(1 + \alpha t)$, where l_0 and l_t are the length of the wire at 0° C. and t° C. respectively and α is the coefficient of expansion. It is peculiar that in several cases the difference between the observed and calculated values was far larger at the ordinary laboratory temperature than at any other.

Below are given the complete set of observations on the first wire tested, viz., soft iron. For the other metals I give merely the value found for the coefficient of expansion and the average difference between the observed and calculated lengths at the various temperatures.

TABLE XII.—Soft Iron.

Temperature.	Length of wire in centimetres.		Observed ∽ Calculated.
	Observed.	Calculated.	
°C.			
15·54	78·40940	78·40987	·00047
24·57	78·41789	78·41779	·00010
33·51	78·42567	78·42563	·00004
44·07	78·43506	78·43489	·00017
54·03	78·44341	78·44362	·00021
65·82	78·45398	78·45396	·00002
75·41	78·46248	78·46236	·00012
88·06	78·47376	78·47345	·00031

For the observations recorded in Table XII. the coefficient of expansion for soft iron is ·00001118. Another determination gave $\alpha =$ ·00001111. The mean of these, $\alpha =$ ·000011145, is the value used to correct the observed periods of oscillation in the determination of the temperature coefficient of the rigidity modulus.

The coefficients of expansion of the other wires experimented on and also of the gun-metal bar already referred to are contained in Table XIII.

TABLE XIII.

Metal.	Coefficient of expansion (α) (mean of 2, 3 or 4 determinations).	Average difference between the observed lengths at the various temperatures and those calculated from the formula $l_t = l_0 (1 + \alpha t)$.
		centims.
Copper	·00001685	·00044
Commercial copper	·00001677	·00004
Steel	·00001160	·00005
Platinum	·00000901	·00013
Gold	·00001364	·00009
Silver	·00001917	·00018
Aluminium	·00002265	·00017
Gun-metal	·00001737	·00027

I cannot conclude an account of this work without expressing my sincere thanks for the help I have received from Professor POYNTING. The research was entered upon at his suggestion and continued for some months under his supervision, and all through I have had the benefit of his encouragement and advice. I am also indebted to him for the privilege of working in the Physical Laboratory of the University of Birmingham during the Cambridge vacations.

I wish also to acknowledge my indebtedness and sincere thanks to Professor THOMSON for his kind encouragement and valuable suggestions during the course of the experiments at the Cavendish Laboratory.

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